When Do Coevolutionary Algorithms Exhibit Evolutionary Dynamics?

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Overview

- Preliminaries
- The Basic Question
- Objective Dynamical Equivalence
- Beyond Objective Dynamical Equivalence

Model and Assumptions

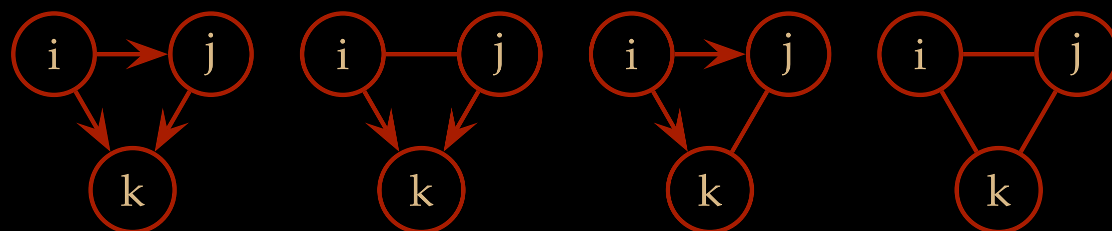
- Rank-Based Selection
  - $\mathcal{G} = \mathcal{M} \circ \mathcal{S} \circ \mathcal{R}$
  - Truncation Selection, Tournament Selection, Rank Selection, etc.
- Complete Mixing
  - $\vec{u} = A\vec{x}$
  - (genome utilities = payoff matrix $\times$ peer proportions)
- Either Finite or Infinite Populations
- Either Finite or Infinite numbers of different genomes

Ranking Functions $\mathcal{R}$

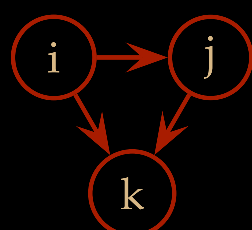
- Objective
  - Always establishes the same ordering among all individuals in an infinite population regardless of the proportions in the population vector $\vec{x}$.
  - ... $\mathcal{R}_i \gtreqless \mathcal{R}_j$ invariant even when $x_i = 0, x_j = 0$
- Existentially Objective
  - Always establishes the same ordering among individuals in an infinite population which have non-zero proportions in $\vec{x}$
  - ... $\mathcal{R}_i \gtreqless \mathcal{R}_j$ consistent only when $x_i \neq 0, x_j \neq 0$
  - Objective $\Longrightarrow$ Existentially Objective

The Payoff Matrix A

- Weakly Transitive
  - For distinct i, j, k: $$(A_{ij} > A_{ji} \wedge A_{jk} \geq A_{kj} \longrightarrow A_{ik} > A_{ki}) \wedge$$ $$(A_{ij} = A_{ji} \wedge A_{jk} > A_{kj} \longrightarrow A_{ik} > A_{ki}) \wedge$$ $$(A_{ij} = A_{ji} \wedge A_{jk} = A_{kj} \longrightarrow A_{ik} = A_{ki})$$



- Strongly Transitive
  - Weakly Transitive and $A_{ij} \neq A_{ji}, A_{jk} \neq A_{kj}, A_{ik} \neq A_{ki}$



Rank Equivalence

- Fitness measures f and g are rank equivalent if they order the genotypes in the same way.
  - There exists an ordering of f and of g such that $\forall j, k$ genotypes, $(f_j > f_k \longleftrightarrow g_j > g_k)$
  - $f \equiv_R g$
- When is a coevolutionary fitness measure rank equivalent to some Objective or Existentially Objective EA fitness measure?

Guaranteeing Rank Equivalence

- Why guarantee that a CEA subjective measure is Rank Equivalent to some [existentially-] objective EA measure?

- Dynamic equivalence to an EA
  - "Safe in the Knowledge" that the CEA is "evolving" in the sense that an EA does
  - (Both Objective and Existentially Objective)
- Context-independent objective measure
  - Comparing algorithm performance
  - Detecting arms races
  - Validating the external measure itself
  - (Objective only)

Transitivity Is Insufficient

- Weak / Strongly Transitive ranking function cannot guarantee rank equivalence to an [Existentially] Objective EA fitness measure.
  - $A = \begin{bmatrix} 0 & 6 & 3 \\ -6 & 0 & 18 \\ -3 & -18 & 0 \end{bmatrix} \qquad \vec{x} = \begin{bmatrix} x_i \\ x_j \\ x_k \end{bmatrix}$
  - $x_i \triangleright x_j \triangleright x_k$
  - $\vec{x} = [\frac{1}{2} \quad \frac{1}{2} \quad 0]^T \quad \Longrightarrow \quad \vec{u} = A\vec{x} = [3 \quad -3 \quad \frac{-21}{2}]$
  - $\vec{x} = [\frac{1}{3} \quad \frac{1}{3} \quad \frac{1}{3}]^T \quad \Longrightarrow \quad \vec{u} = A\vec{x} = [3 \quad 4 \quad \frac{-21}{3}]$

Objective Rank Equivalence

- To show CEA rank equivalence to an objective measure:
  - Must show that $\forall i, j : (f_i > f_j \longleftrightarrow \forall k : A_{ik} > A_{jk})$
  - ...because *entire* population context could consist solely of a single genome k
- Consequences:
  - **If:** an objective measure exists which is rank equivalent to the subjective fitness measure of the CEA
  - **Then:** All possible genomes may be assigned a context-free "fitness" according to that ranking
  - **And Thus:** there exists an EA (using that "fitness") which is dynamically equivalent to the CEA

For Example . . .

- The *greater than game*
  - Individual i beats individual j when $A_{ij} > A_{ji}$
  - e.g., i receives scaled difference between the payoffs
- More generally: when subjective utility is linearly related to some objective measure, $u \sim_L f$
  - $A_{ij} = \alpha f_i + \beta f_j$ (as long as $\alpha > 0$)
  - ... the CEA using A for fitness is dynamically equivalent to an EA using f for fitness

Existentially Objective Rank Equivalence

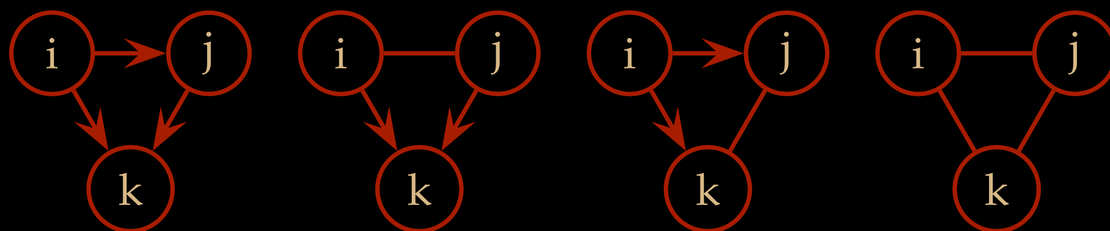
- When is a CEA rank equivalent to an Existentially Objective EA ranking function?
  - One necessary condition:

$$\forall i,j,k : (\mathcal{R}_i > \mathcal{R}_j \longrightarrow (A_{ij} > A_{jj} \wedge A_{ii} \geq A_{ji}) \vee (A_{ij} \geq A_{jj} \wedge A_{ii} > A_{ji})) \quad \wedge$$
$$(\mathcal{R}_i = \mathcal{R}_j \longrightarrow A_{ik} = A_{jk})$$

- Consequences
  - Ordering within the population will always be the same between the CEA and some EA
  - CEA will be dynamically equivalent to the EA
  - Does *not* guarantee that we may can *determine* the context-free rank of an individual.

A Sufficient Condition

- A CEA is dynamically equivalent to an EA if payoff matrix A is:
  - Weakly Transitive...



  - Monotone... $\forall i, j, k$ (not necessarily distinct):
    $A_{ij} \geq A_{ji} \wedge A_{jk} \geq A_{kj} \longrightarrow A_{ik} \geq \max(A_{ij}, A_{jk})$
  - Constant-Sum Plus... (Constant Sum, with slightly looser restrictions along the diagonal of A)

Example

- GP Checkers Game
  - Infinite kinds of players
  - All players play all other players (including themselves)
  - Winner of match gets +1, loser gets -1, draws get 0
  - Weak Transitivity (such as: if A beats B, and B beats C, then A will beat C, etc.)
- ...then no reason to play all those games!
  - Just do tournament selection, and do lazy evaluation by directly playing A against B when in the tournament.

Conclusions

Recap

- Assumptions
  - Complete Mixing, Rank-Based Selection
- We show:
  - Transitivity does not guarantee dynamic equivalence to an EA
  - Necessary and sufficient conditions to guarantee a true objective function for full-mixing CEA
  - Necessary conditions for guaranteeing dynamic equivalence to EA
  - An example giving sufficient conditions

Where To Go From Here?

- Permitting Intransitivity
  - Discovering “local” intransitive cycles and path-compressing them into transitive DAGs
- Experimental Analysis
  - How does deviating from the minimum conditions given affect the effectiveness of the CEA?
  - How does deviating from full mixing affect the dynamics of the CEA?

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