Data Reliability Scoring

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Outline

- What is data reliability score?
- How can we define data reliability?
- Gram Determinant score and main result
- Experiments

Data reliability

POLITICS | POLICY

Trump Orders Firing of Bureau of Labor Statistics Chief

The president's move throws the quality of America's statistical apparatus into question

By *Justin Lahart* Follow, *Alex Leary* Follow and *Matt Grossman* Follow

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Data reliability

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Data misreporting during the COVID19 crisis

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Perspect Health Inf Manag. 2021 Oct 1;18(4):1f.

UPCODING MEDICARE: IS HEALTHCARE FRAUD AND ABUSE INCREASING?

Alberto Coustasse

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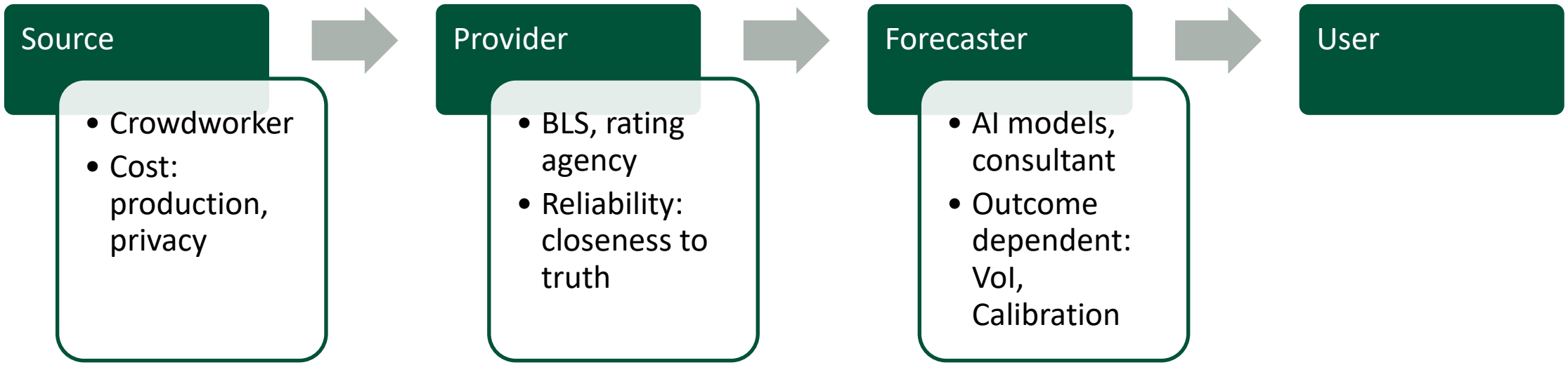
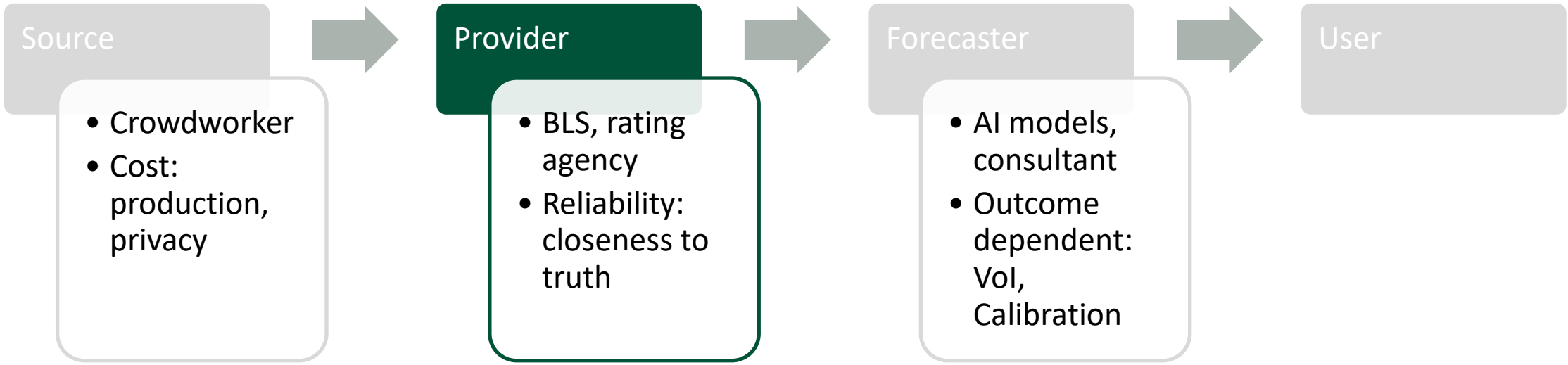
Scores for Data



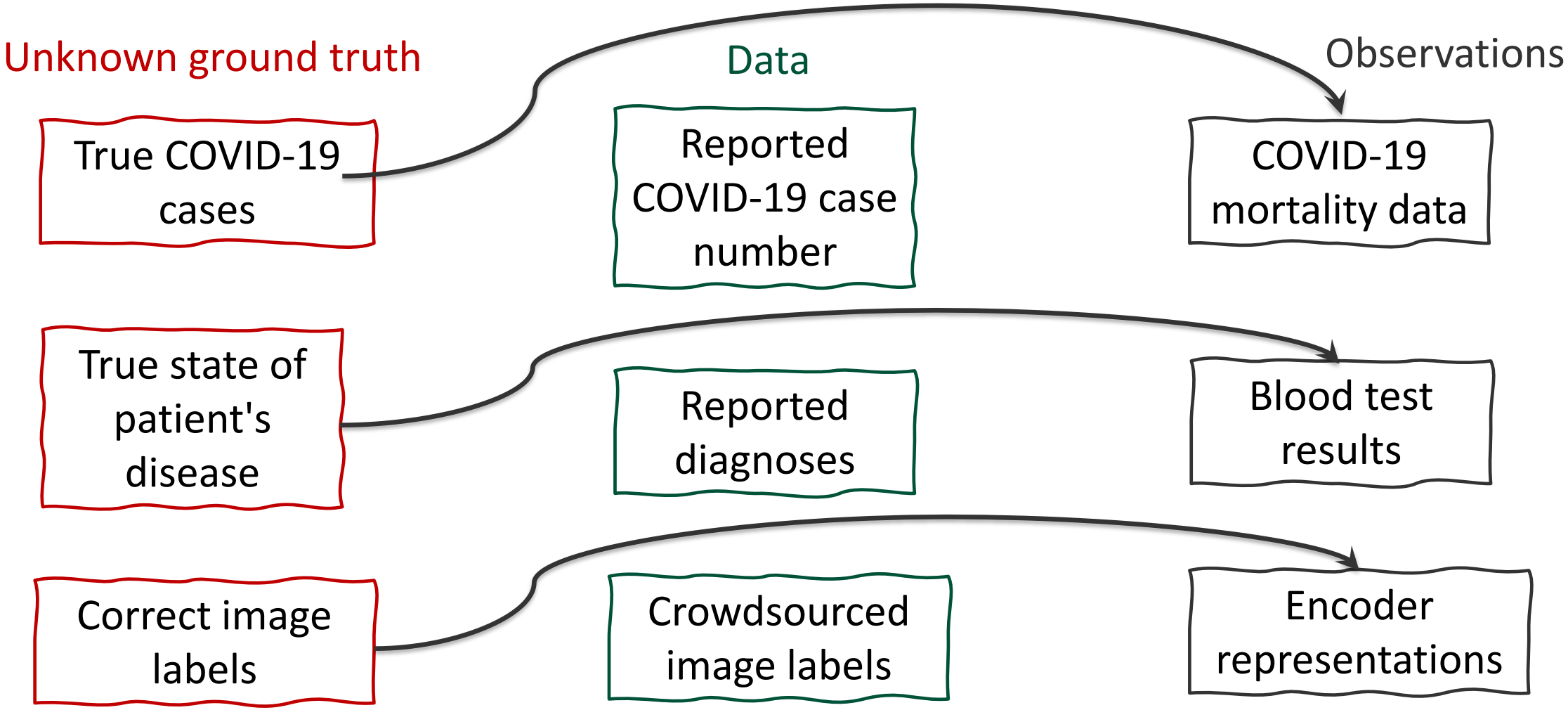
Data reliability score



Why difficult?

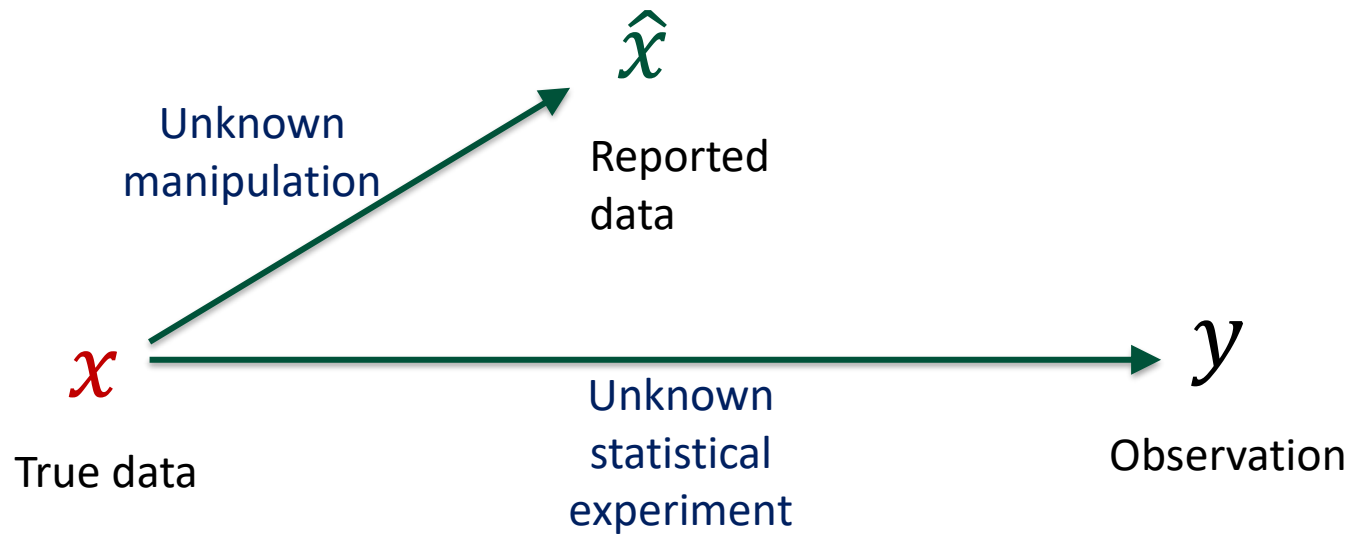
Unknown ground truth	Data
True COVID-19 cases	Reported COVID-19 case number
True state of patient's disease	Reported diagnoses
Correct image labels	Crowdsourced image labels

Why promising?



Research Question

- Can we design a reliability score $S(\hat{x}, y)$ that assigns a higher expected score to datasets that are closer to the ground truth?



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- Gram determinant score and main result
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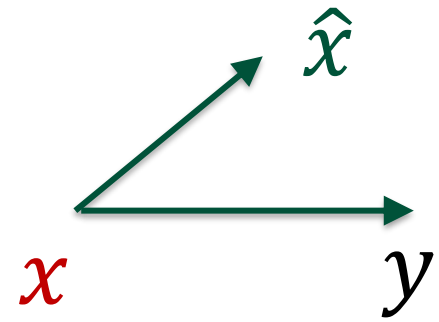
Reliability Ordering of Datasets

- To compare $\hat{x}$ and $\hat{x}'$ w.r.t. true data x, we propose partial orderings $>^{\cdot}$
  - Exact match ordering: $\hat{x} >^{x}_{EXACT} \hat{x}'$ if $\hat{x} = x$ but $\hat{x}' \neq x$



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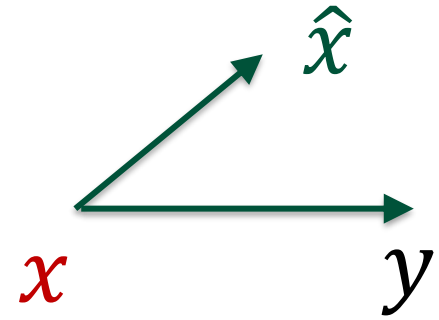
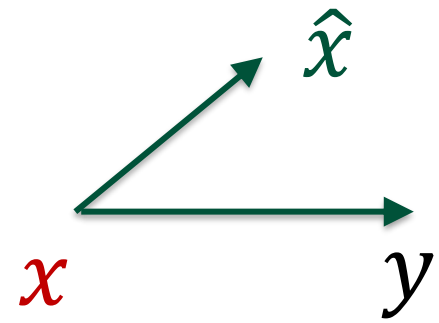
  - Blackwell dominant ordering: one is a garbling of the other
  - Hamming ordering: $\hat{x}$ is closer to x than $\hat{x}'$ in Hamming distance



Scoring to Preserve Reliability Ordering

- To compare $\hat{x}$ and $\hat{x}'$ w.r.t. true data x, we propose partial orderings $>^{\cdot}$
  - Exact match ordering
  - Blackwell dominant
  - Hamming ordering
- A reliability score S preserves $>^{\cdot}$ if

$$\mathbb{E}[S(\hat{x}, y)] > \mathbb{E}[S(\hat{x}', y)] \text{ for all } \hat{x} >^{x} \hat{x}'$$



Problem Formulation

Design reliability score S preserves $>^{\cdot}$ so that

$$\mathbb{E}[S(\hat{x}, y)] > \mathbb{E}[S(\hat{x}', y)] \text{for all } \hat{x} >^{x} \hat{x}'$$

- Three datasets
  - (Hidden) true data $x = (x_1, \ldots, x_N) \in [d]^N$
  - Reports $\hat{x} = (\hat{x}_1, \ldots, \hat{x}_N) \in [d]^N$
  - Observation $y = (y_1, \ldots, y_N) \in \mathcal{Y}^N$

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- Assumption: $y_n \sim P(\cdot, x_n)$ independently for all i

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Main Result

We introduce **Gram Determinant score (GDS)**

$$\det(\hat{G})$$

where $\hat{G}$ is the Gram matrix of report and observation

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$$\det(\hat{G})$$

which preserves

- Exact match ordering under $\mathcal{P}_{indep}$ and $\mathcal{Q}_{nonperm}$
- Blackwell ordering under $\mathcal{P}_{indep}$ and $\mathcal{Q}_{reg}$
- $\frac{1}{4L}$ −Hamming ordering under $\mathcal{P}_{indep}$ and $\mathcal{Q}_{L,K}$

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- What is data reliability?
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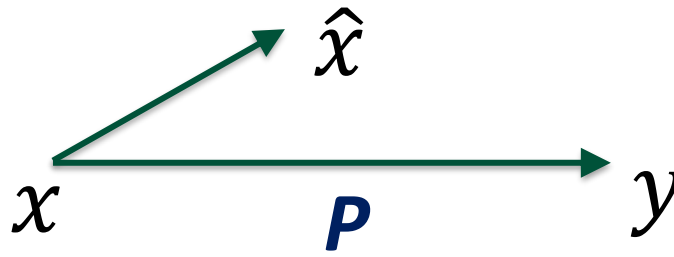
  Proof idea: GDS preserves Exact match
- Experiments

GDS=volume of $\hat{x}$, y dist.

- Given three datasets x, $\hat{x}$, y,
  - Experiment $P \in \mathbb{R}^{y \times d}$ independently for all i
  - Misreport matrix $Q \in \mathbb{R}^{d \times d}$: $Q(i,j) = \frac{\#[x_n = i, \hat{x}_n = j]}{N}$
  - Joint distribution and $\hat{x}$ and y: $PQ = PQ_x Q_{\hat{x}|x}^T$
  - Gram matrix of $\hat{x}$ and y: $\hat{G} = (PQ)^T PQ$



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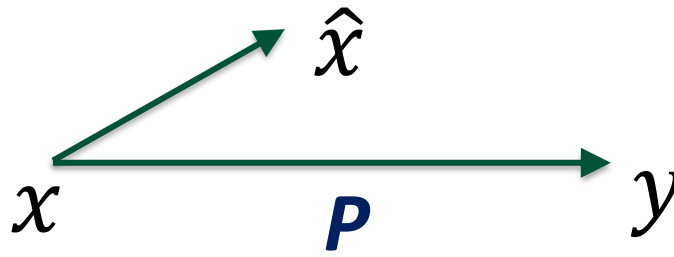

- GDS $\det(\hat{G}) = vol(PQ)$ volume spanned by the joint distribution

GDS preserves Exact match

Claim: $\mathbb{E}[S(\hat{x}', y)] < \mathbb{E}[S(x, y)]$for all $\hat{x} \neq x$

- Three facts for $vol(A) \coloneqq \det A^T A$
  1. $\det AB = \det A \det B$ and $\det B = \det B^T$
  2. $\det A = \prod eign_i(A)$
  3. eigenvalue of stochastic matrix≤ 1

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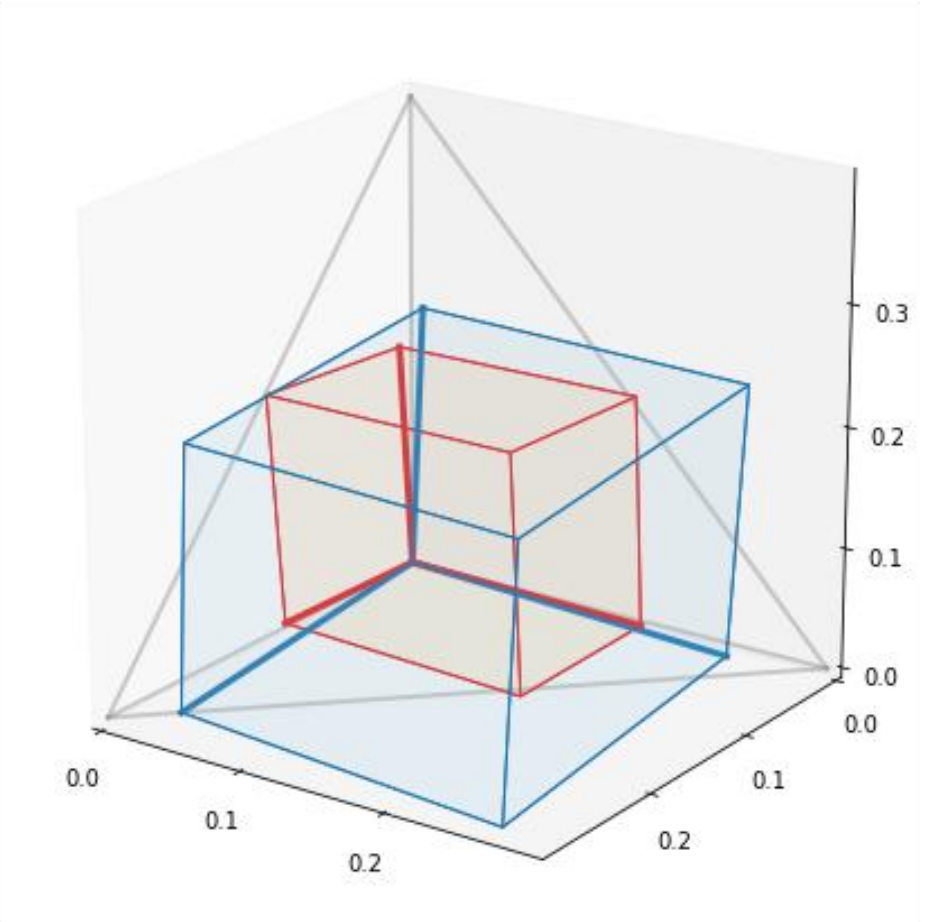
  3. eigenvalue of stochastic matrix ≤ 1
- Proof:

$$\mathbb{E}[S(\hat{x}, y)] = vol\left(PQ_x Q_{\hat{x}|x}^T\right)$$
$$= vol(PQ_x)vol\left(Q_{\hat{x}|x}\right)$$
$$\leq vol(PQ_x) = \mathbb{E}[S(x, y)]$$



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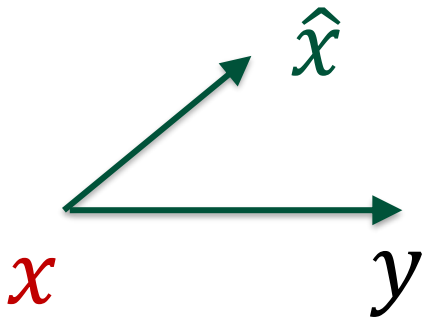
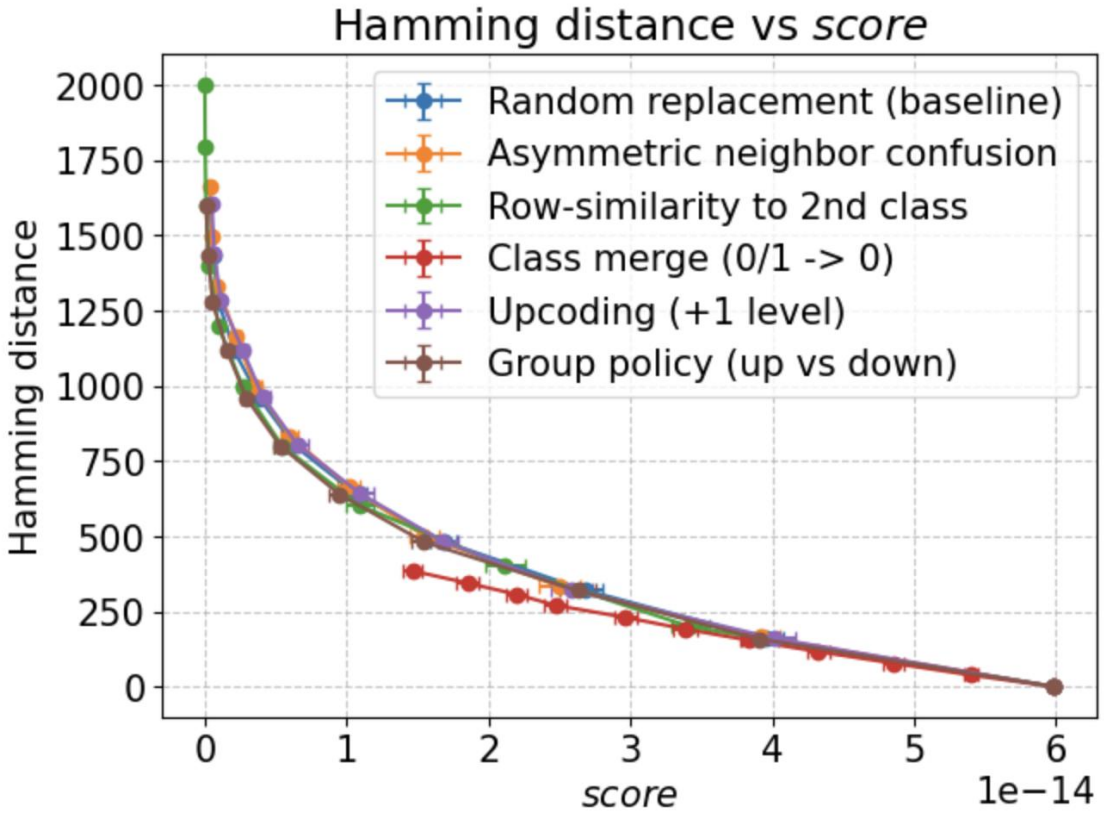
Empirical Results – Synthetic Data

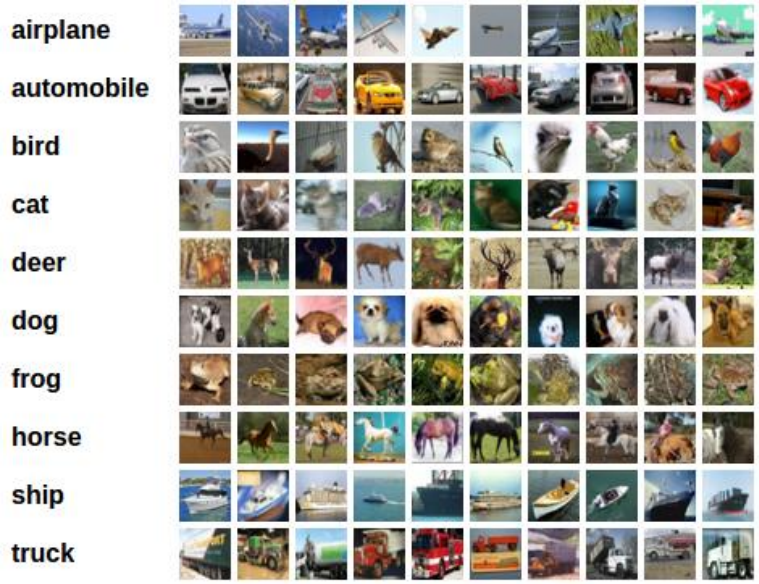
- $x \sim \{0,1,2,3,4\}^N$ with $N = 4000$
- $\hat{x}$: noisy label
- y: random experiment $P \in [0,1]^{5\times5}$

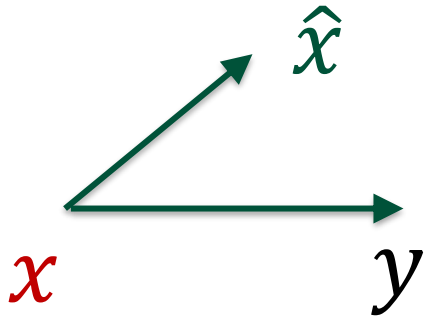


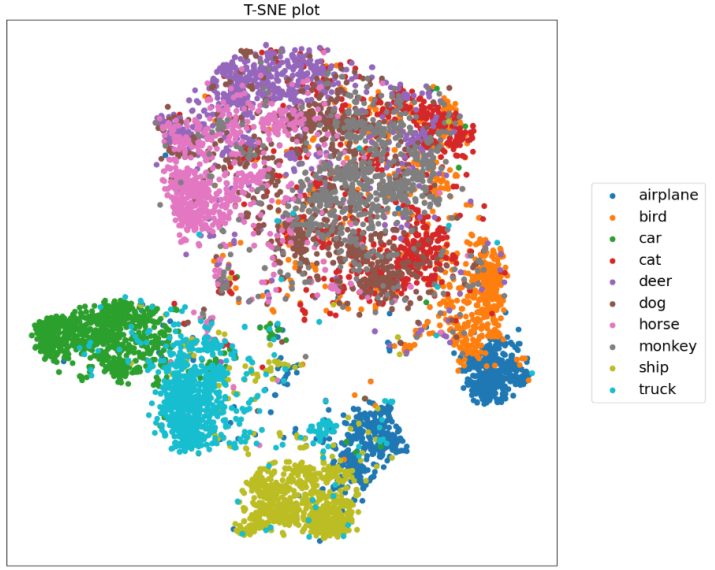


Empirical Results – CIFAR-10 data

- x: real label of image
- $\hat{x}$: noisy label
- y: feature embedding from contrastive learning







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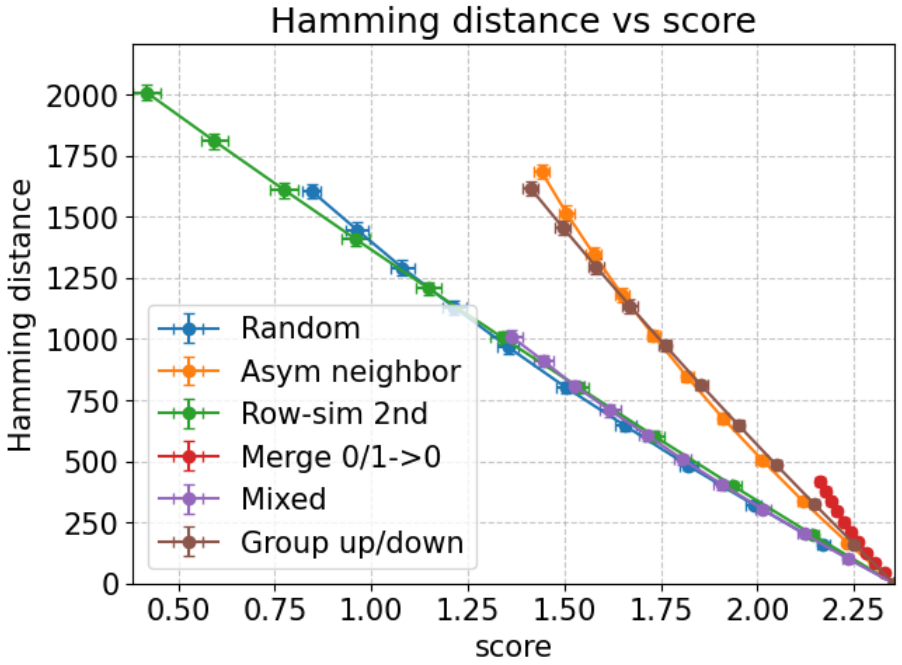
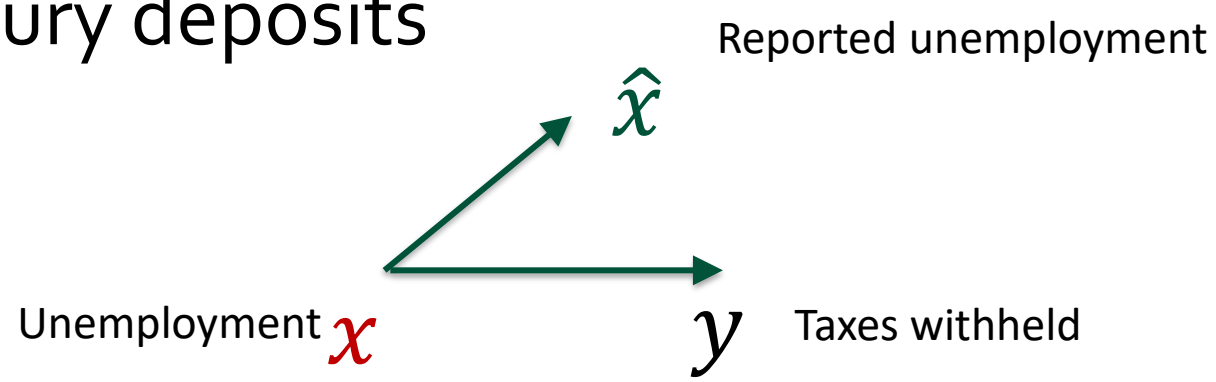


Unemployment data

- x: real unemployment number
- $\hat{x}$: reports
  - First report,
  - One month revision, and
  - Final revision
- y: Withheld Taxes from Treasury deposits



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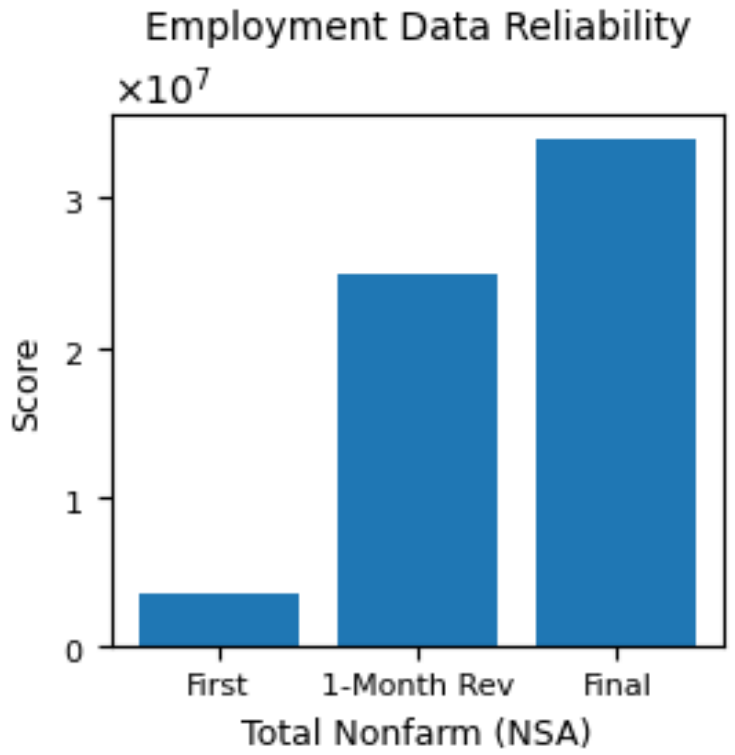

Discussion

- Summary
  - Reliability ordering
  - GDS: respects several orderings and agonistic to experiment
  - Kernel treat for high dimensional Y
- Future directions:
  - How to score data without verification?
    - Time series data, or large signal space X
    - ordering vs testing
  - Empirical results