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We still assume Alice and Bob share a key.
Intuitively, we want a way to "mark" or "tag" a message, using the secret key.
The recipient of the message should be able to verify that the tag was generated using the shared key: it could only have come from Bob.

# Message Authentication

## Message Authentication Codes

A MAC consists of three p.p.t. algorithms, $(\mathsf{Gen}, \mathsf{Mac}, \mathsf{Vrfy})$.

$\mathsf{Gen}(1^n)$: outputs a key $k$, $|k| > n$.

$\mathsf{Mac}(k, m)$: outputs a tag, $t$. This might be randomized, so we write it as $t \leftarrow \mathsf{Mac}(k, m)$. Also, $m$ might be variable length, or we might only support some fixed length for each $n$: $m \in \{0,1\}^{\ell(n)}$

$\mathsf{Vrfy}(k, m, t)$: is a deterministic algorithm that outputs a bit $b$. $b = 1$ means $t$ is a valid tag for $m$ using $k$, and $b = 0$ means $t$ is invalid and $m$ should be rejected.

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Just as with encryption, we have a correctness requirement:
For every $n$, every $k$ output by Gen and every $m$ (of appropriate size),
$\mathsf{Vrfy}(k, m, \mathsf{Mac}(k, m)) = 1$.

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Canonical Verification: Unlike with encryption schemes, deterministic MAC algorithms *can* be secure. For such algorithms, we have a simple way of verifying.
$\mathsf{Vrfy}(k, m, t)$: Compute $t' = \mathsf{Mac}(k, m)$. If $t' = t$ output 1. Else, output 0.

# Unforgeability

We define a new security game, Mac-forge$_{\mathcal{A},\Pi}$, for capturing that a MAC scheme, $\Pi = (\mathsf{Gen}, \mathsf{Mac}, \mathsf{Vrfy})$ is *unforgeable*. That is, without the key $k$, an adversary $\mathcal{A}$ cannot create a valid tag.

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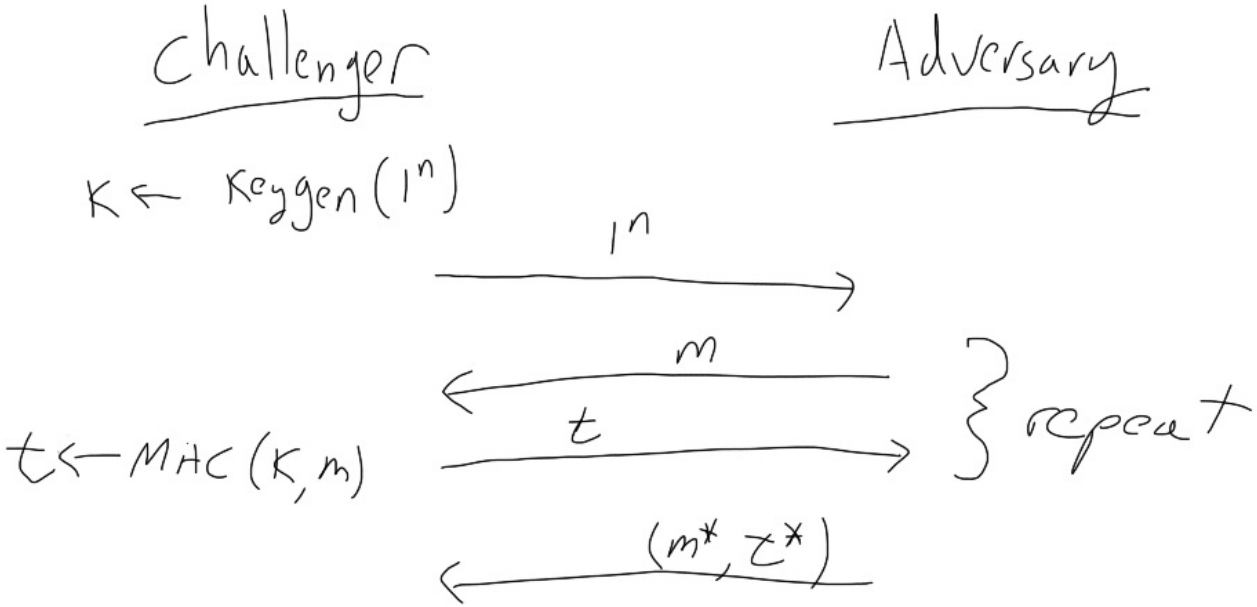


Let $Q$ be the set of messages queried

$\mathcal{A}$ wins if 1) $\text{Vrfy}(k, m^*, t^*) = 1$

2) $m^* \notin Q$

# Unforgeability

## Definition

A MAC scheme $\Pi = (\mathrm{Gen}, \mathrm{Mac}, \mathrm{Vrfy})$ is existentially unforgeable under an adaptive chosen-message attack, (or, *secure*) if for all p.p.t. adversaries $\mathcal{A}$, there is some negligible function negl such that:

$$\Pr[\text{Mac-forge}_{\mathcal{A},\Pi}(n) = 1] \leq \mathrm{negl}(n)$$

# A Construction

## A fixed length MAC, $\Pi$

Let $F$ be a PRF. We define a fixed length MAC for messages of length $n$:

$\text{Gen}(1^n) : k \leftarrow \{0,1\}^n$

$\text{Mac}(k, m)$: If $|m| \neq n$, abort. Otherwise, output $t = F_k(m)$.

$\text{Vrfy}(k, m, t)$: If $|m| \neq n$, output 0. Otherwise, compute $t' = F_k(m)$. If $t' = t$ output 1, otherwise, output 0.

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Claim: If there exists $\mathcal{A}$ with non-negligible advantage in the Mac-forge$_{\mathcal{A},\Pi}$ game, there exists $\mathcal{A}_r$ that wins in the $\text{PrivK}^{\text{prf}}_{\mathcal{A},\text{F}}$ game with probability $\frac{1}{2} + \frac{1}{p(n)}$.