El Gamal Encryption

Slides by Prof. Jonathan Katz.
Lightly edited by me.

Dlog-based PKE

Diffie-Hellman key exchange

$\xrightarrow{G,\ q,\ g,\ h_1}$

$\xleftarrow{h_2}$

$\xleftarrow{c_2 = \ k \cdot m}$

$(G, q, g) \leftarrow \mathcal{G}(1^n)$

$x \leftarrow \mathbb{Z}_q$

$h_1 = g^x$

$k = (h_2)^x$

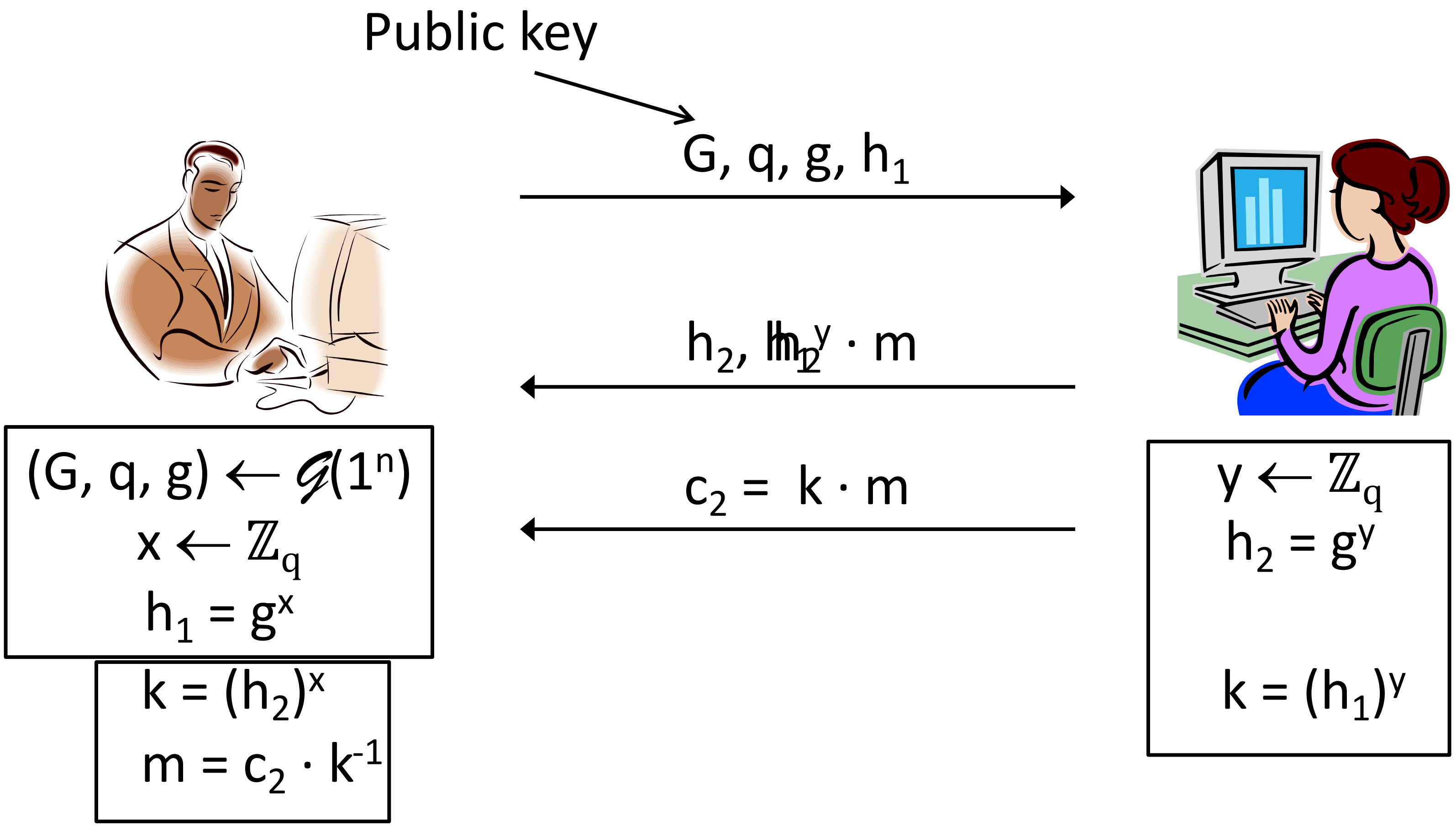
$m = c_2 \cdot k^{-1}$

$y \leftarrow \mathbb{Z}_q$

$h_2 = g^y$

$k = (h_1)^y$

El Gamal encryption



Public key

G, q, g, h_1

$h_2, h_1^y \cdot m$

$c_2 = k \cdot m$

$(G, q, g) \leftarrow \mathcal{G}(1^n)$

$x \leftarrow \mathbb{Z}_q$

$h_1 = g^x$

$k = (h_2)^x$

$m = c_2 \cdot k^{-1}$

$y \leftarrow \mathbb{Z}_q$

$h_2 = g^y$

$k = (h_1)^y$

El Gamal encryption

- Gen(1^n)
  - Run $\mathcal{G}(1^n)$ to obtain G, q, g. Choose uniform $x \in \mathbb{Z}_{q.}$ The public key is (G, q, g, g^x) and the private key is x

- $\text{Enc}_{pk}(m)$, where pk = (G, q, g, h) and $m \in G$
  - Choose uniform $y \in \mathbb{Z}_{q.}$ The ciphertext is g^y, $h^y \cdot m$

- $\text{Dec}_{sk}(c_1, c_2)$, where sk = x
  - Output $c_2/c_1^x = c_2 \cdot c_1^{-x}$

Security?

- If the DDH assumption is hard for $\mathcal{G}$, then the El Gamal encryption scheme is CPA-secure
  - Follows from security of Diffie-Hellman key exchange, or can be proved directly
  - Note that the discrete-logarithm assumption alone is not enough here

$\Rightarrow$ Secure for encryption of multiple messages (using the same public key)!

- Note that sender(s) must use fresh randomness for each encryption

El Gamal in practice

- Parameters G, q, g are standardized and shared

- Need to encode message as a group element
  - In some groups, there are natural ways to do this
  - In other cases, not as easy
  - Will see later a better way of resolving this issue

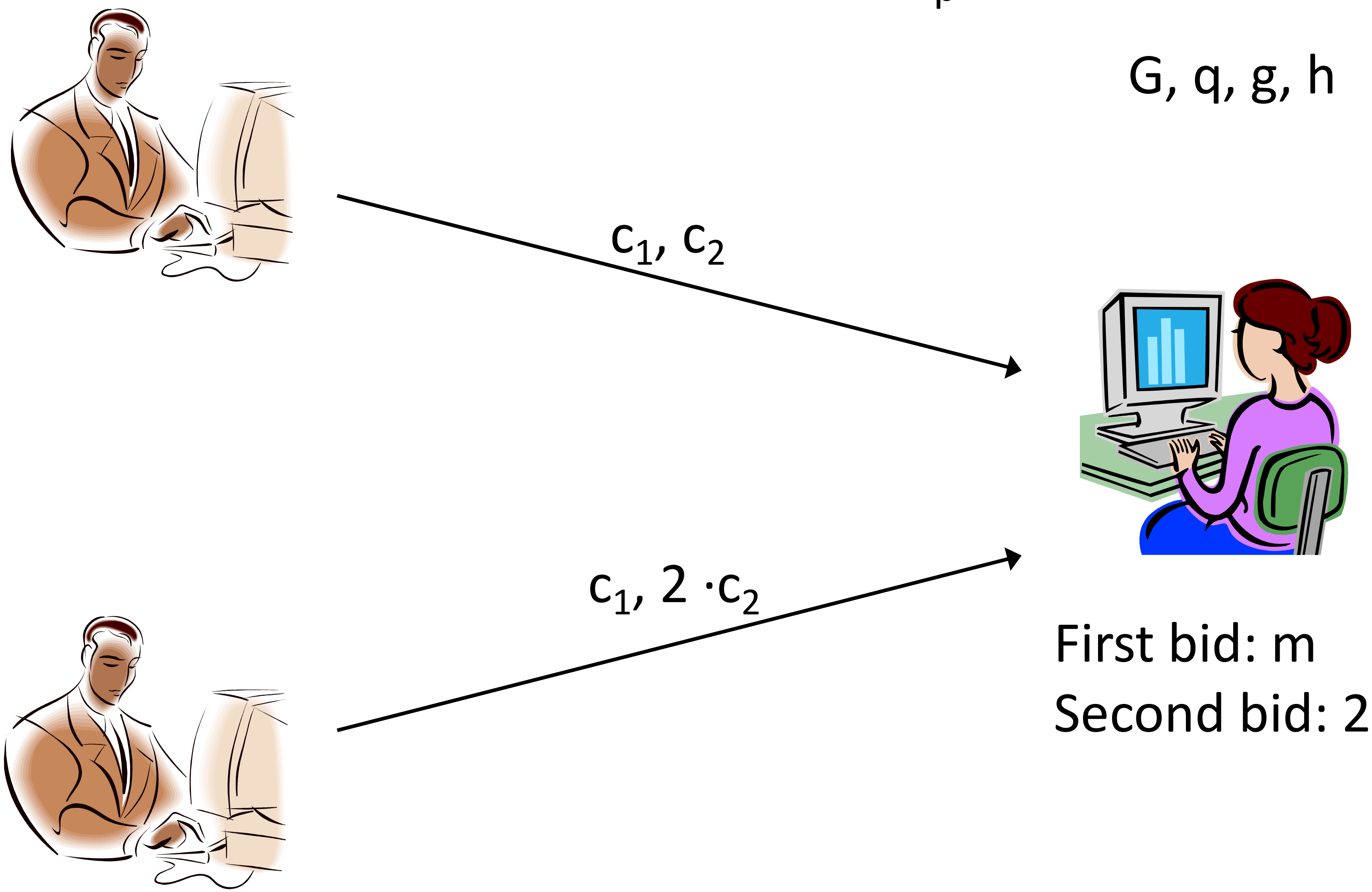
Chosen-ciphertext attacks?

- El Gamal encryption is *not* secure against chosen-ciphertext attacks
  - Follows from the fact that it is *malleable*

- Given ciphertext (c_1, c_2), transform it to obtain the ciphertext $(c_1, c'_2) = (c_1, \alpha \cdot c_2)$ for arbitrary α
  - Since $(c_1, c_2) = (g^y, h^y \cdot m)$,
    we have $(c_1, c'_2) = (g^y, h^y \cdot (\alpha m))$
  - I.e., encryption of m becomes an encryption of αm!

Attack!

(Assume $2 \in G \subset \mathbb{Z}^*_p$)



G, q, g, h

First bid: m

Second bid: 2m