Key Exchange

and the Public Key Revolution

Slides by Prof. Jonathan Katz.
Lightly edited by me.

Private-key cryptography

- Private-key cryptography allows two users who *share a secret key* to establish a “secure channel”

- The need to share a secret key has several drawbacks…

The key-distribution problem

- *How do users share a key in the first place?*
  - Need to share the key using a secure channel…
- This problem can be solved in some settings
  - E.g., physical proximity, trusted courier, …
  - Note: this does not make private-key cryptography useless!
- Can be difficult, expensive, or impossible to solve in other settings

The key-management problem

- Imagine an organization with N employees, where each pair of employees might need to communicate securely

- Solution using private-key cryptography:
  - Each user shares a key with all other users

  $\Rightarrow$ Each user must store/manage N-1 secret keys!

  $\Rightarrow$ $O(N^2)$ keys overall!

Key Distribution Centers

Drawbacks:

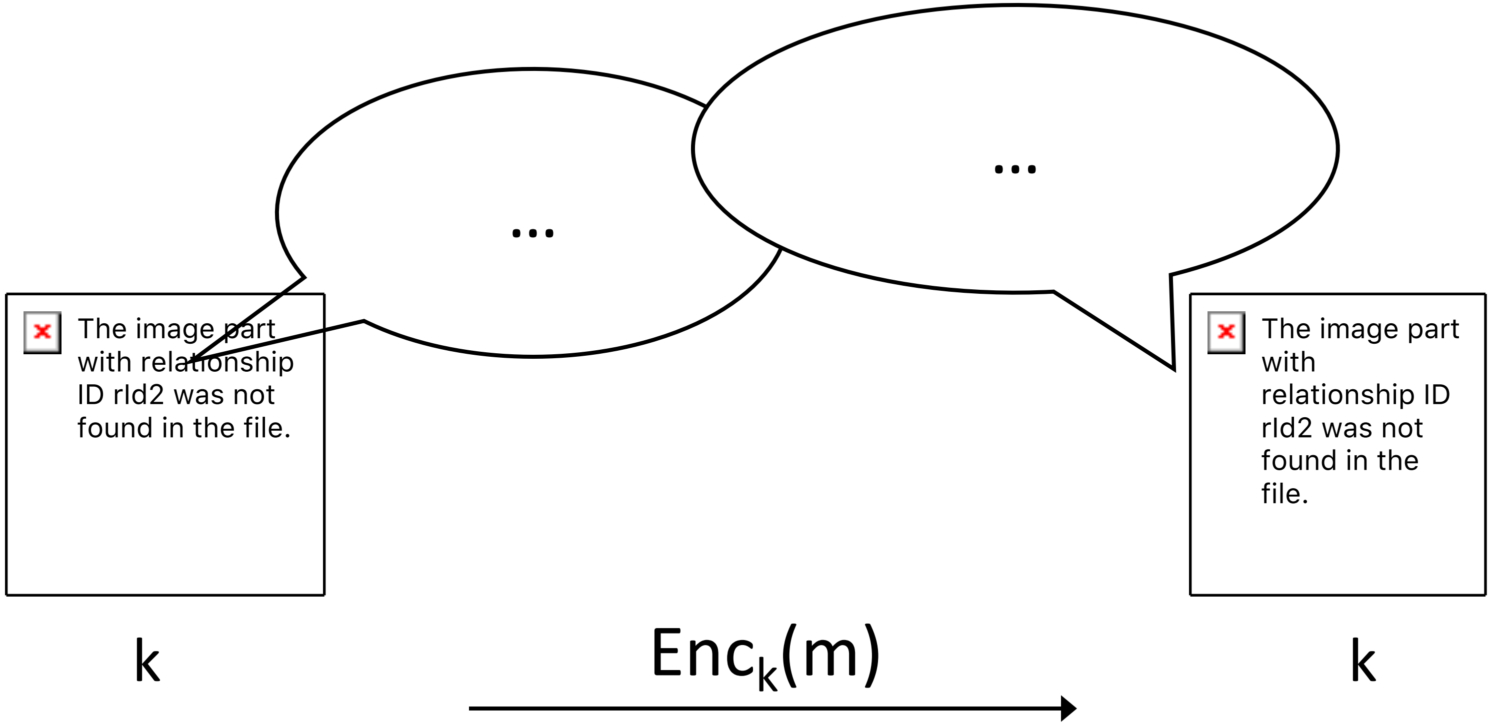
- Single point of failure.
  - For liveness.   Could duplicate, but…
  - For security! Internal and external.
- Cannot support "open systems".
  - What if Alice and Bob do not work for the same entity, or trust the same person?
  - E.g. sending credit card information to a merchant.

“Classical” cryptography
offers no solution
to these problems!

New directions…

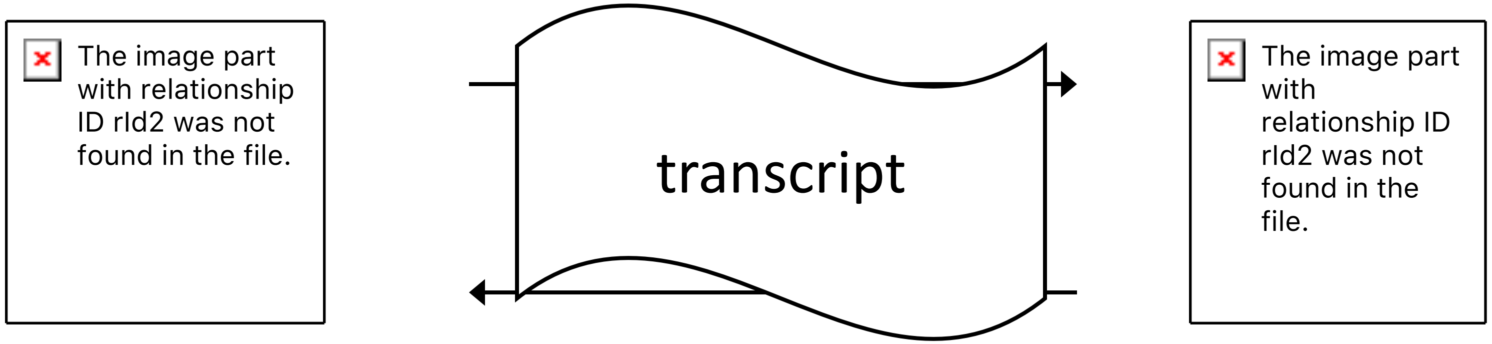
- Main ideas:
  - Some problems exhibit *asymmetry* – easy to compute, but hard to invert (factoring, RSA, group exponentiation, …)
  - Use this asymmetry to enable two parties to agree on a shared secret key via public discussion(!)
    - *Key exchange*

Key exchange



Secure against an eavesdropper who sees everything!

More formally…



$k \in \{0,1\}^n$ $k \in \{0,1\}^n$

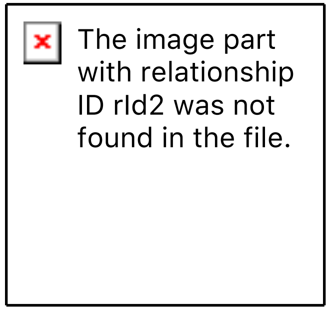
<u>Security goal</u>: even after observing the transcript, the shared key k should be indistinguishable from a uniform key

Notes

- Being unable to <u>compute</u> the key given the transcript is not a strong enough guarantee

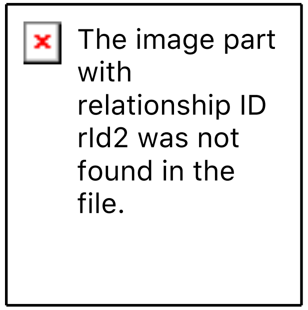
- Indistinguishability of the shared key from uniform is a <u>much</u> stronger guarantee…
  - …and is necessary if the shared key will subsequently be used for private-key crypto!

Diffie-Hellman key exchange



G, q, g, h_1 →

← h_2



$k_1 = (h_2)^x$

$k_2 = (h_1)^y$

$(G, q, g) \leftarrow \mathcal{G}(1^n)$

$x \leftarrow \mathbb{Z}_q$

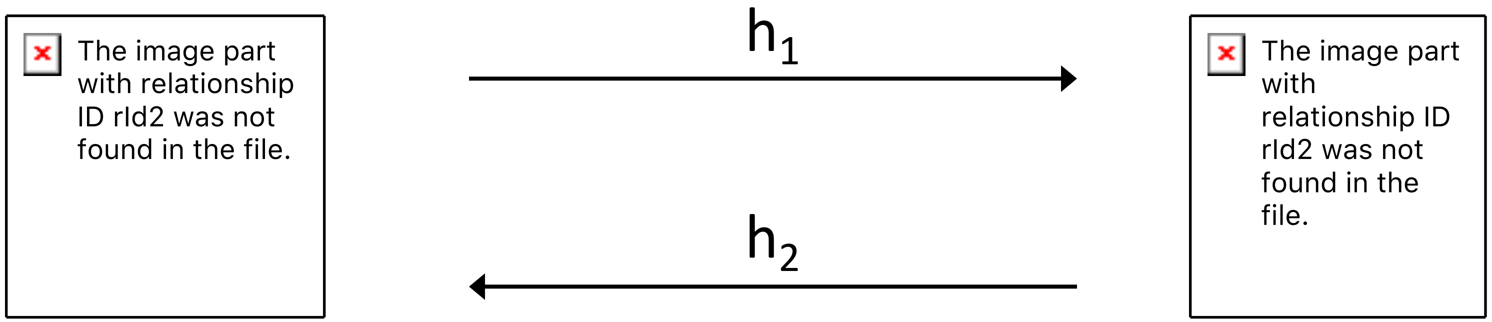
$h_1 = g^x$

$y \leftarrow \mathbb{Z}_q$

$h_2 = g^y$

In practice…

G, q, g



$k_1 = (h_2)^x$

$k_2 = (h_1)^y$

$h_1 = g^x$

$h_2 = g^y$

Recall…

- *Decisional Diffie-Hellman (DDH) assumption:*
  - Given g^x, g^y, cannot distinguish g^{xy} from a uniform group element

Security?

- Eavesdropper sees G, q, g, g^x, g^y
- Shared key k is g^{xy}

- Computing k from the transcript is exactly the *computational* Diffie-Hellman problem

- Distinguishing k from a uniform group element is exactly the *decisional* Diffie-Hellman problem
  - $\Rightarrow$ If the DDH problem is hard relative to $\mathcal{G}$, this is a secure key-exchange protocol!

A subtlety

- We want our key-exchange protocol to give us a uniform(-looking) key $k \in \{0,1\}^n$
- Instead we have a uniform(-looking) group element $k \in G$
  - Not clear how to use this as, e.g., an AES key
- Solution: *key derivation*
  - Set k' = H(k) for suitable hash function H
    - Secure if H is modeled as a random oracle

Modern key-exchange protocols

- Security against passive eavesdroppers is insufficient
- Generally want *authenticated* key exchange
  - This requires some form of setup in advance

- Modern key-exchange protocols provide this
  - We will return to this later