

# Defining Public Key Encryption

Slides by Prof. Jonathan Katz.  
Lightly edited by me.

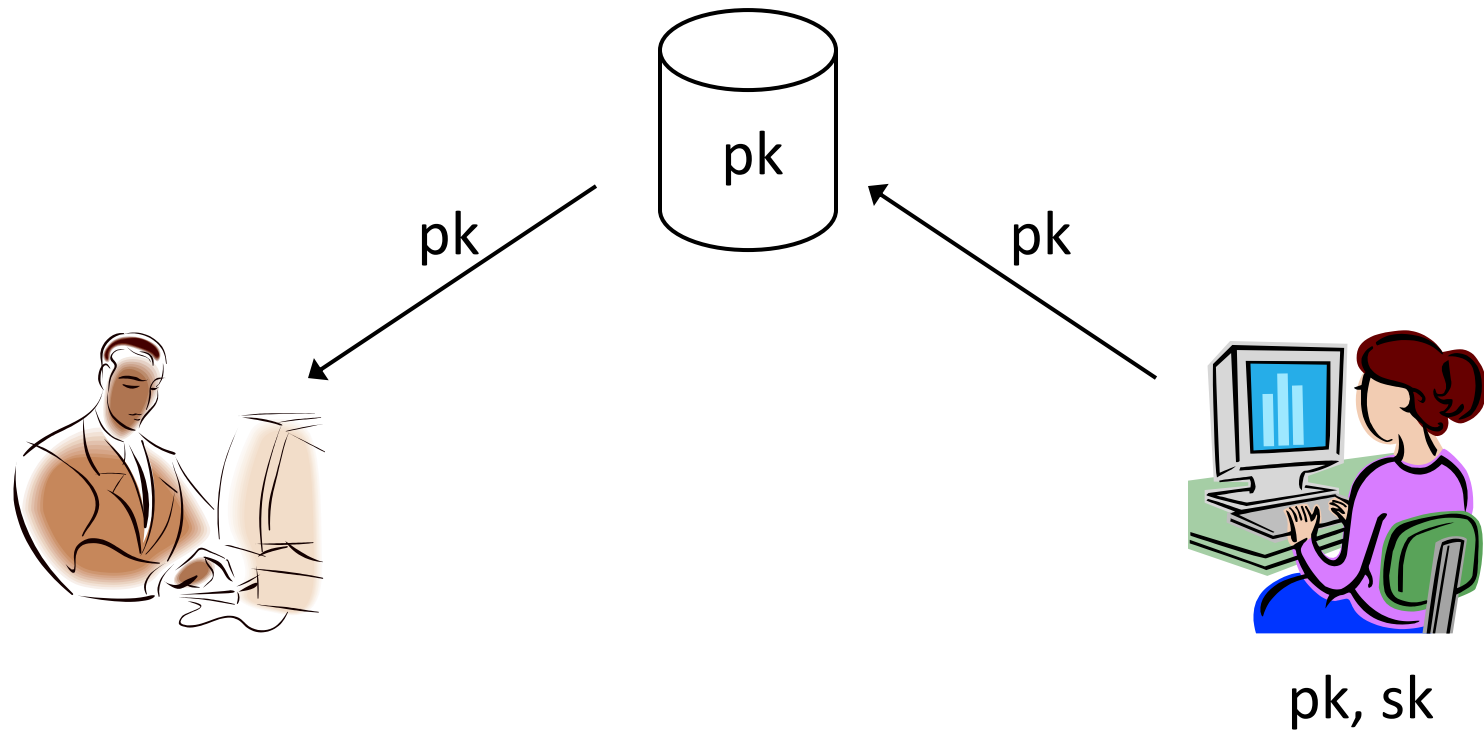
# Review: private-key setting

- Two (or more) parties who wish to securely communicate *share* a uniform, secret key  $k$  in advance
- Same key  $k$  used for sending or receiving
  - Either party can send or receive
  - If multiple parties share a key, no way to distinguish them from one another based on the key
- Secrecy of  $k$  is critical
  - No security if attacker knows  $k$

# The public-key setting

- One party generates a *pair* of keys:  
public key  $pk$  and private key  $sk$ 
  - Public key is widely disseminated
  - Private key is kept secret, and shared with no one
- Private key used by the party who generated it;  
public key used by anyone else
  - Also called *asymmetric* cryptography
- Security must hold even if the attacker knows  $pk$

# Public-key distribution I



# Public-key distribution II



← pk



pk, sk

# Public-key distribution

- Previous figures (implicitly) assume parties are able to obtain correct copies of each others' public keys
  - I.e., the attacker is *passive* during key distribution
- We will revisit this assumption later

# Primitives

	<b>Private-key setting</b>	<b>Public-key setting</b>
<b>Secrecy</b>	Private-key encryption	Public-key encryption
<b>Integrity</b>	Message authentication codes	Digital signature schemes

# How does this address the drawbacks of private-key crypto...?

- Key distribution
  - Public keys can be distributed over *public* (but authenticated) channels
- Key management in system of N users
  - Each user stores 1 private key and N-1 *public keys*; only N keys overall
  - Public keys can be stored in a central, public directory
- Applicability to “open systems”
  - Even parties who have no prior relationship can find each others' public keys and use them



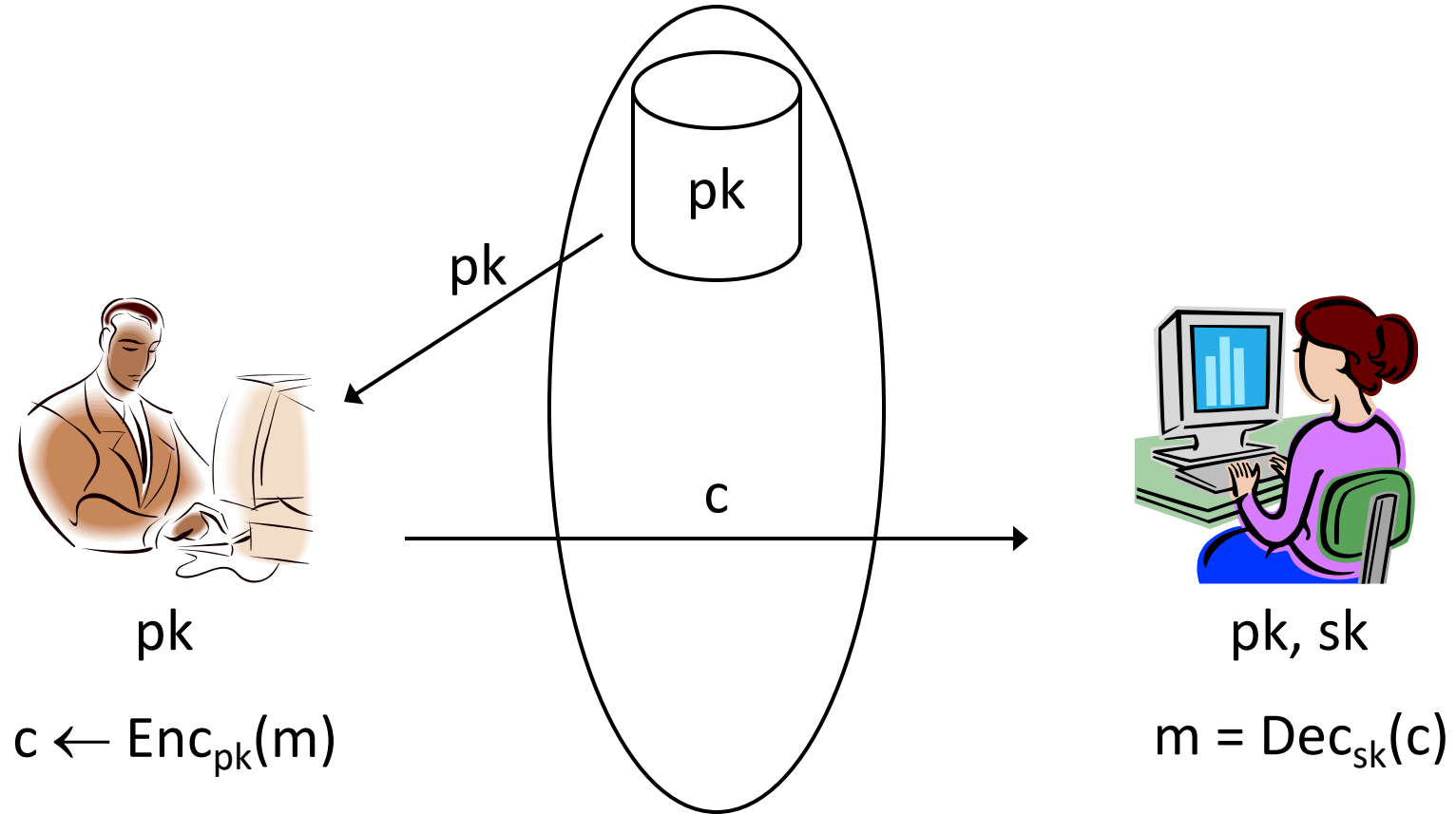
# Public-key vs. private-key crypto

- Public-key cryptography is *strictly stronger* than private-key cryptography
  - Parties who wish to securely communicate could simply each generate public/private keys and then share them with each other
  - Use appropriate key depending on who is sending or receiving

# Why study private-key crypto?

- Public-key crypto is roughly 2-3 orders of magnitude *slower* than private-key crypto
- Also 2-10× higher communication
  - If private-key crypto is an option, better to use it!
- As we will see, private-key cryptography is used for efficiency even in the public-key setting

# Public-key encryption



# Public-key encryption

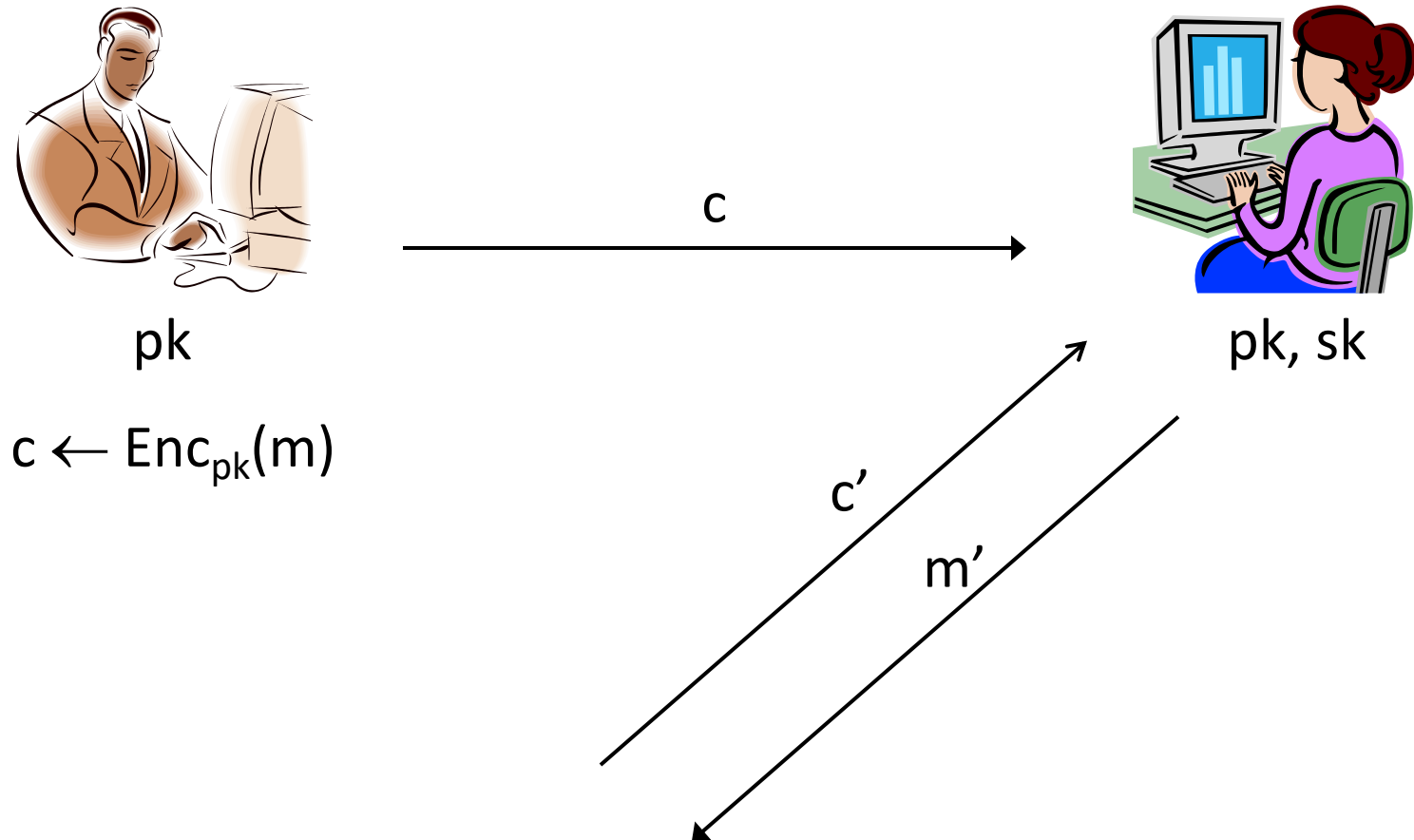
- A public-key encryption scheme consists of three PPT algorithms:
  - Gen: *key-generation algorithm* that on input  $1^n$  outputs  $(pk, sk)$
  - Enc: *encryption algorithm* that on input  $pk$  and a message  $m$  outputs a ciphertext  $c$
  - Dec: *decryption algorithm* that on input  $sk$  and a ciphertext  $c$  outputs a message  $m$  or an error  $\perp$

For all  $m$  and  $(pk, sk)$  output by Gen,  
$$\text{Dec}_{sk}(\text{Enc}_{pk}(m)) = m$$

# Notes on the definition

- No encryption oracle?!
    - Encryption oracle redundant in public-key setting
- ⇒ No *perfectly secret* public-key encryption
- ⇒ No *deterministic* public-key encryption scheme can be CPA-secure
- ⇒ CPA-security implies security for encrypting multiple messages (as in the private-key case)

# Chosen-ciphertext attacks



# Chosen-ciphertext attacks

- Chosen-ciphertext attacks are arguably even a greater concern in the public-key setting
  - Attacker might be a legitimate sender
  - Easier for attacker to obtain full decryptions of ciphertexts of its choice
- Related concern: *malleability*
  - I.e., given a ciphertext  $c$  that is the encryption of an unknown message  $m$ , might be possible to produce ciphertext  $c'$  that decrypts to a related message  $m'$
  - This is also undesirable in the public-key setting

# Chosen-ciphertext attacks

- Can define CCA-security for public-key encryption by analogy to the definition for private-key encryption
  - See book for details