Relaxing Security

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We will use a security parameter, n, as input to Gen.
We want to build encryption schemes that:
For any adversary with runtime that increases as poly(n) ,
The adversary's distinguishing advantage is negl(n).

Negligible Functions

A negligible function approaches 0 faster than any inverse polynomial.

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Where do these become smaller than $\frac{1}{n^5}$?

$$2^{-n} < \frac{1}{n^5} \text{ for } n \geq 23$$

$$2^{-\frac{n}{2}} < \frac{1}{n^5} \text{ for } n \geq 59$$

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Note: $2^{-\sqrt{n}} < n^{-\log n}$ for every $n > 65,536$.

Adding Negligible Functions

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Claim: The function $f(x) = \text{negl}_1(x) + \text{negl}_2(x)$ is negligible.

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Let $q(x) = 2p(x)$. Because $\text{negl}_1(x)$ is a negligible function, there exists some N_1 such that $\forall n > N_1, \text{negl}_1(n) < \frac{1}{q(n)}$.

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Define $N = \max(N_1, N_2)$.
$\forall n > N, \text{negl}_1(n) + \text{negl}_2(n) < \frac{2}{q(n)} = \frac{2}{2p(n)} = \frac{1}{p(n)}$.

Multiplying a Negligible Function by a Polynomial

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Consider the polynomial $\frac{q(x)}{p(x)}$. Since negl(x) is negligible, there exists $N \in \mathbb{N}$ such that $\forall n > N, \text{negl(n)} < \frac{\text{q(n)}}{\text{p(n)}}$

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It follows that $\forall n > N, p(n) \cdot \text{negl(n)} = \text{f(n)} < \text{q(n)}$

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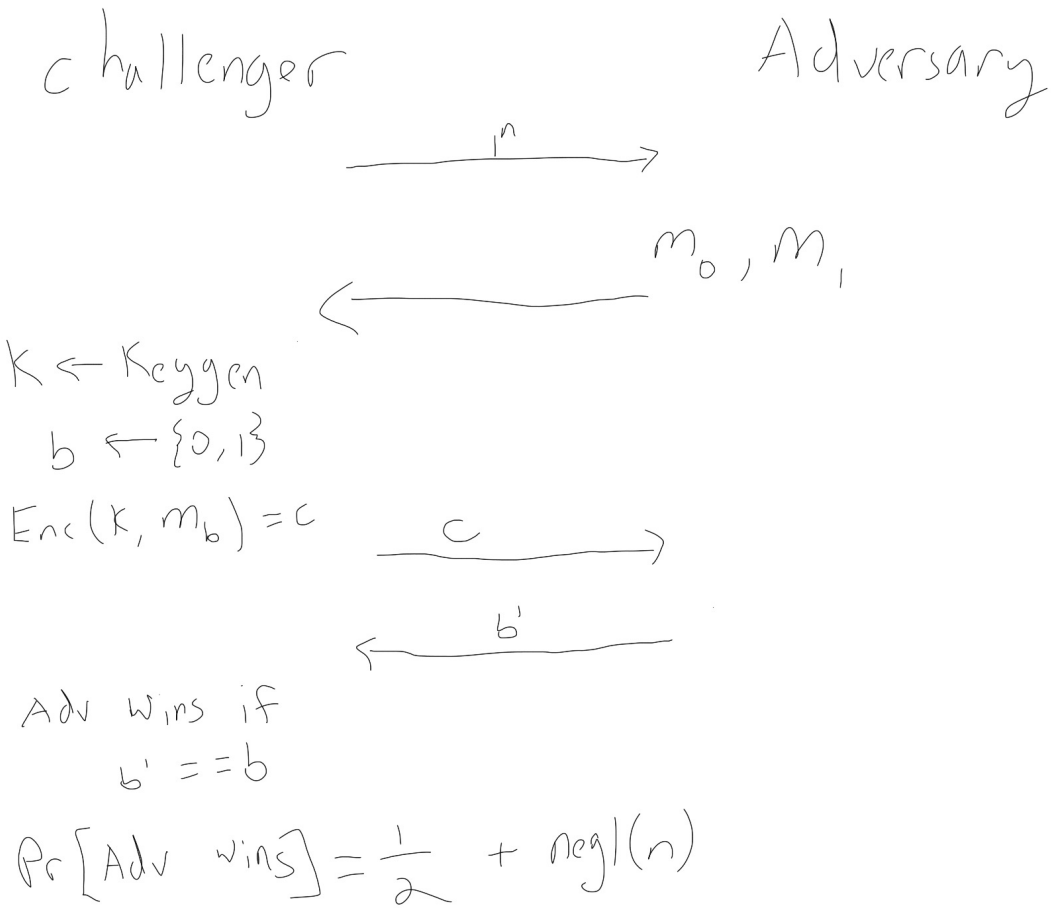

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These relaxations are both necessary:

1. Given ciphertext c, try decrypting c using every key $k \in \mathcal{K}$.
   Denote the resulting set by $M(c)$.
   Since $|\mathcal{K}| < |\mathcal{M}|$, there must be some $m \in \mathcal{M}$ such $m \notin M(c)$.
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   Try all k until you find one that is consistent with those ℓ values. Full key recovery!

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2. A polynomial-time adversary can try a few values of k. If she succeeds, then great, and if not, then she can guess b' at random.
   Small advantage, but non-zero.

Unary security parameters

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Technicality: If we represent our key length (i.e. security parameter) in binary, then the input *length* grows *logarithmically* in the key length.
For example: input 10000000 indicates a 128-bit key.
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We specify our security parameter in unary.
$\text{Gen}(1^n)$ outputs an n-bit key.
Each increase in n allows the adversary's runtime to grow. It still doubles the keyspace, ensuring that its advantage diminishes.

Privacy against eavesdroppers

Indistinguishability in the presence of an eavesdropper:

$\mathsf{PrivK}^{\mathsf{eav}}_{\mathcal{A},\Pi}(n)$:

1. $\mathcal{A}$ is given 1^n and outputs m_0 and m_1 such that $|m_0| = |m_1|$.
2. $k \leftarrow \mathit{Gen}(1^n)$, $b \leftarrow \{0,1\}$, and $c \leftarrow \mathit{Enc}(k, m_b)$.
   Then c is given to $\mathcal{A}$.
3. $\mathcal{A}$ outputs a bit b'.
4. The outcome of the experiment is 1 if $b = b'$ and 0 otherwise.

Definition

A private-key encryption scheme $\Pi = (\mathit{Gen}, \mathit{Enc}, \mathit{Dec})$ has indistinguishable encryptions in the presence of an eavesdropper, or is EAV-secure, if for all probabilistic polynomial-time adversaries $\mathcal{A}$, there is a negligible function negl such that, for all n,

$$\Pr[\mathsf{PrivK}^{\mathsf{eav}}_{\mathcal{A},\Pi}(n) = 1] \leq 1/2 + \mathrm{negl(n)}$$