Deterministic Finite Automata (DFAs)

Grammars *generate* strings.
Automata *recognize* strings:
Given some input string x, an automata M either outputs

- "accept" if $x \in \mathcal{L}(M)$, or
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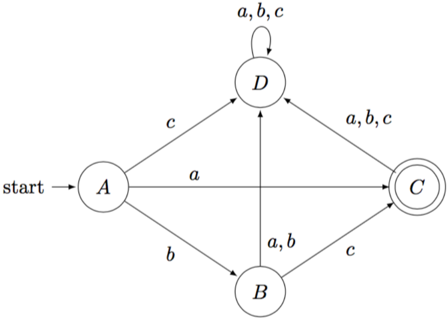

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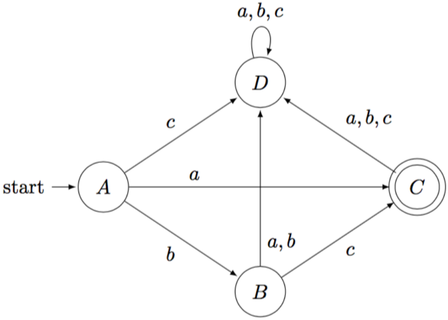

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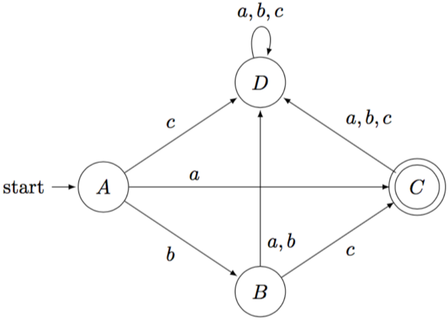

An automata has:

- A set of states.
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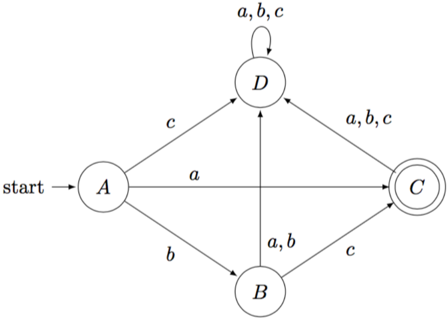
- A set of labeled transitions from one state to another.

DFAs: removing the trap state.



On input x, follow the transitions as you process each character of x, in order.
Accept if and only if you end in an accept state.

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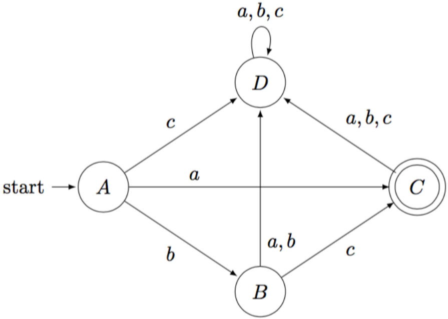

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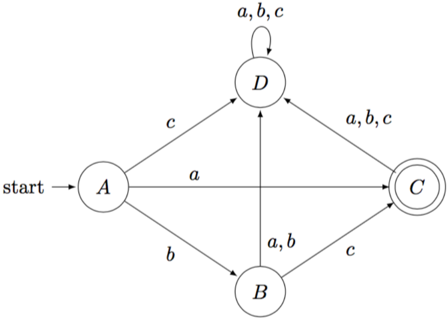

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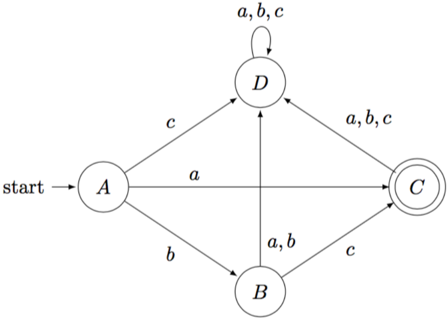

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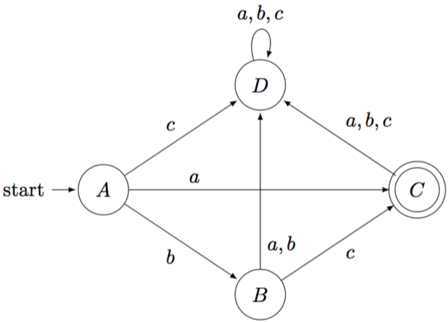

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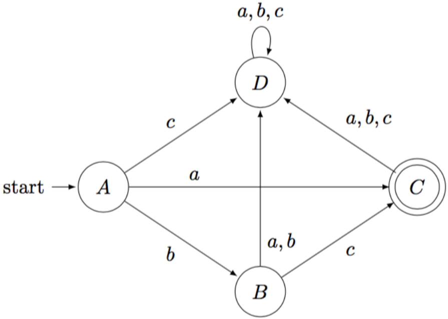

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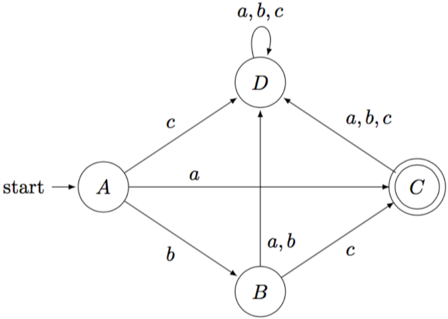

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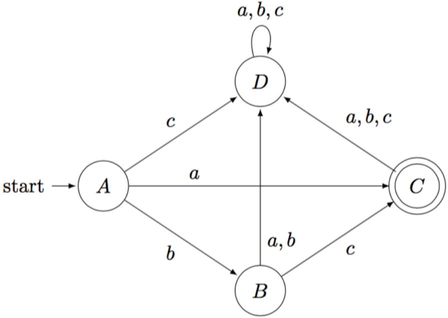
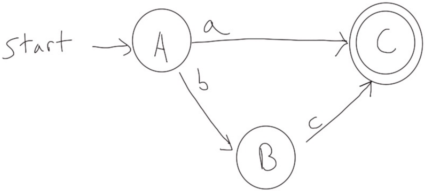
DFAs





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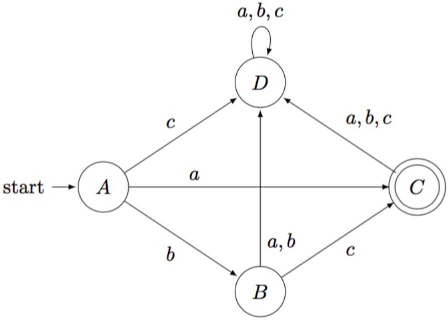

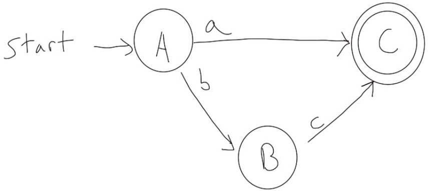

If there is no transition from the current state labeled with the current input character, simply reject.

Example 2

$L = \{x \mid x \in \{a, b\}^* \text{ and every } a \text{ precedes every } b\}$

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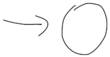

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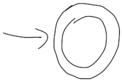

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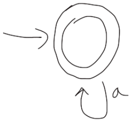

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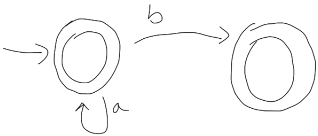

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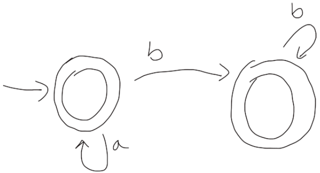

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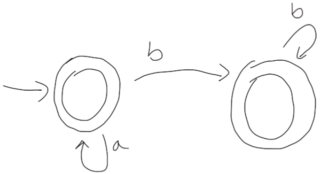

Alternative ways of specifying the same language:
$L = \{x \mid x = yz \wedge y \in \{a\}^* \wedge z \in \{b\}^*\}$

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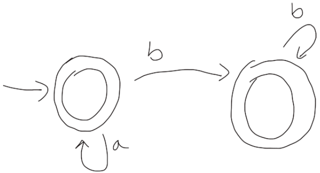

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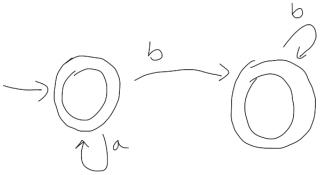

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$L = \{x \mid x \in \{a,b\}^*$ and there is no occurrence of ba in $x\}$

Example 3

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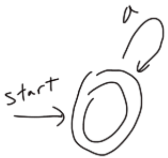

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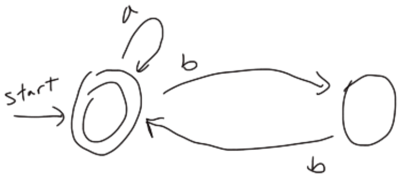

Example 4

L over $\Sigma = \{a, b, c, d\}$ which contains exactly the strings x such that

1. x begins with dc
2. x ends in a substring cd (but $x \neq dcd$) and
3. x has no other occurrence of cd between these 2 substrings.

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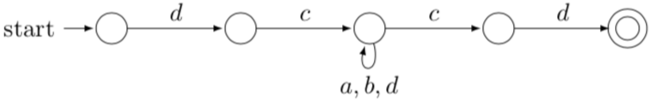

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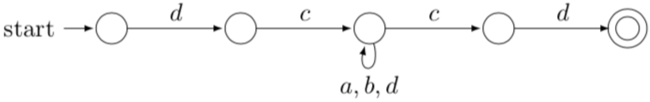

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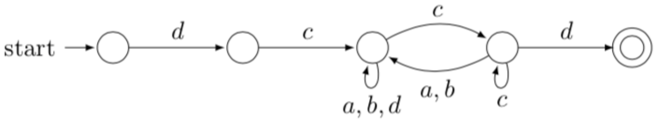

Example 4b

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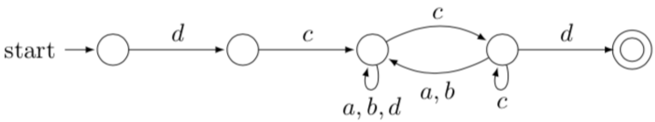

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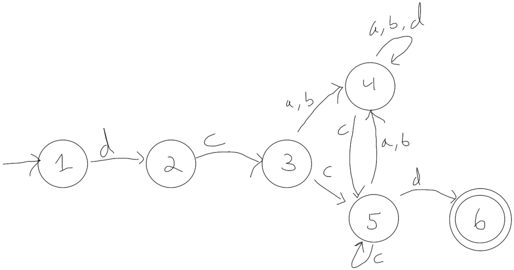

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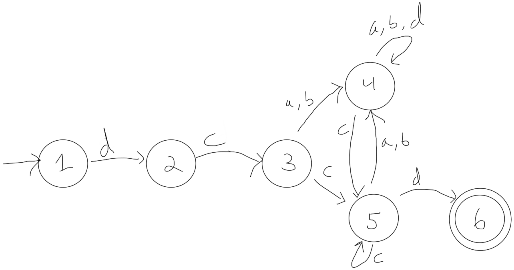

Claim: If $M(x) = 1$, then $x \in L$.

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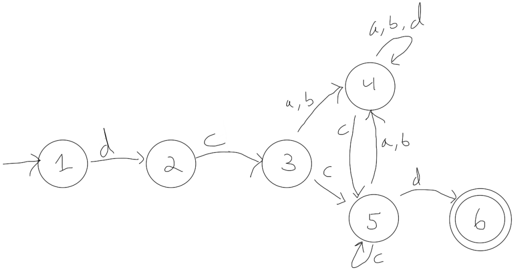

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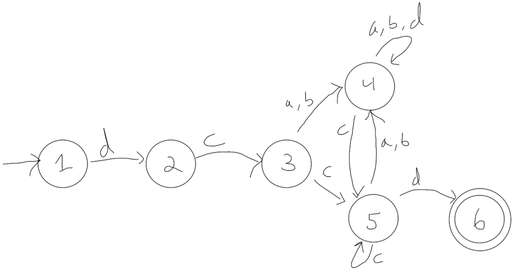

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Property 1: Clearly x starts with dc.
Property 2: It must end with cd, because any string leading to state 5 must end in c: all transitions to 5 are labeled c. We can verify by hand that $M(dcd) = 0$.

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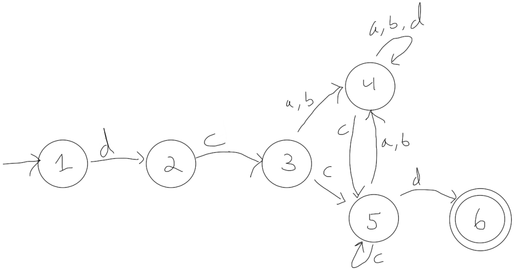

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Property 3: any sub-string ending in c is either in state 3 or 5. Neither of those states have an out-transition labeled d, except the one leading to 6.

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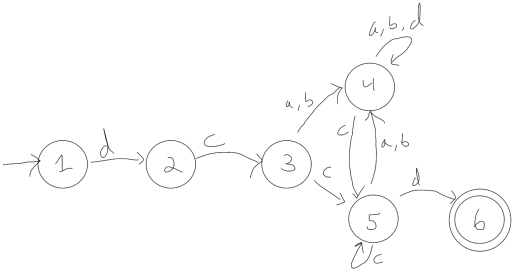

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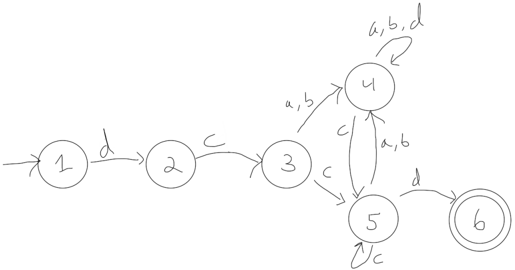

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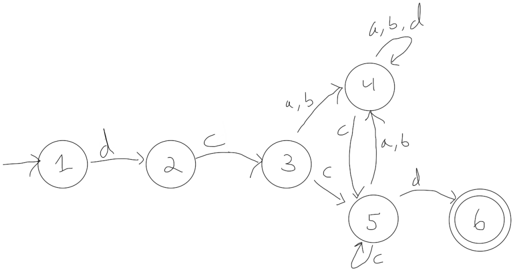

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Let w be such that $x = dcwcd$. The first dc leave us in state 3.

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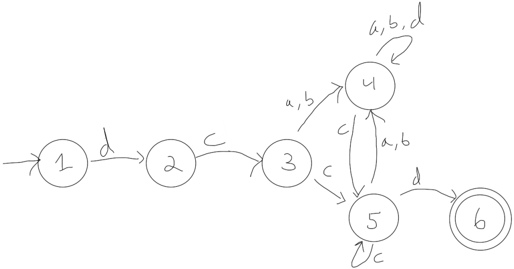

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By the definition of L, w cannot start with a d, so x is not rejected when M is in state 3. Also, M will never return to state 3.

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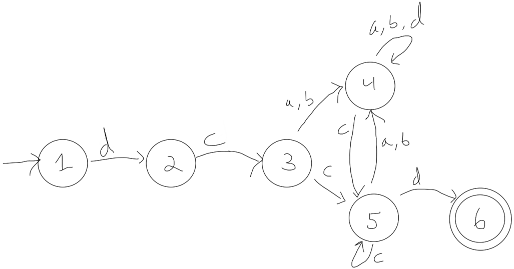

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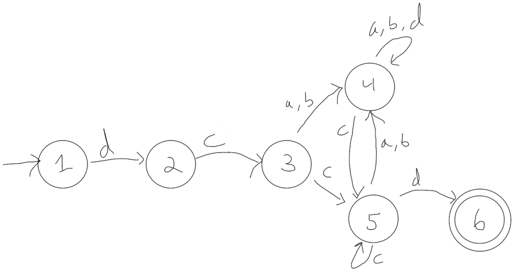

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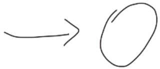
The other two states in the triangle have 4 transitions out, so neither causes a rejection on w.
Finally, since we know that x ends in cd, regardless of where in the triangle w leaves us (even if $w = \Lambda$), cd carries us to the accept state.

Example 5

$$(ab)^* + c^*$$

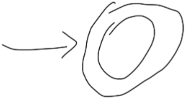
Example 5

$$(ab)^* + c^*$$



Example 5

$$(ab)^* + c^*$$



Example 5

$$\boxed{(ab)^* + c^*}$$

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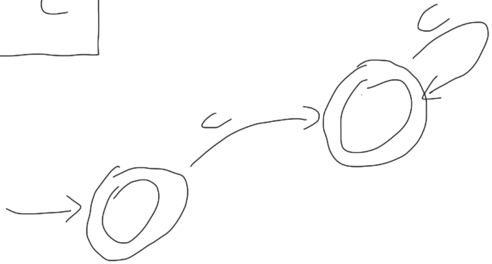
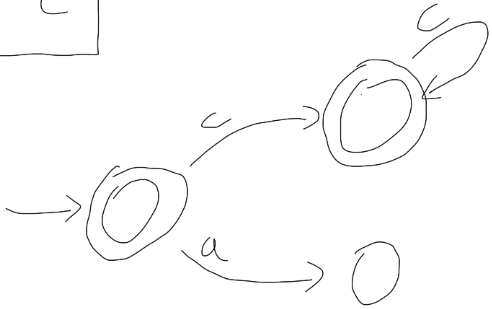


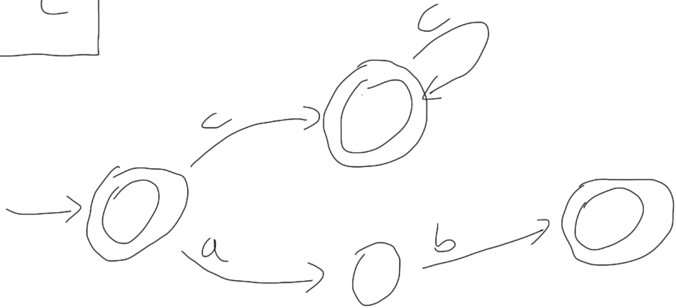
Example 5

$\boxed{(ab)^* + c^*}$



Example 5

$$\boxed{(ab)^* + c^*}$$



Example 5

$(ab)^* + c^*$