CS600	May 5, 2018

Homework 5

Students are welcome to work together, but *every student must write up their own solutions, independently!* I strongly encourage students to use LaTex for writing up their solutions. Please see the course web-page for a template file.

Question 1: What class or classes does the following language belong to? Demonstrate it.
$L = \{\phi \mid \phi \text{ is a boolean formula with exactly 2 solutions}\}$

Question 2: A NAND gate is a Boolean function on 2 input bits that is equal to the NOT of the AND of the two inputs. Here is the truth table for the function.

	0	1
0	1	1
1	1	0

Prove that NAND is complete for the basis B_0. That is, show that all of the Boolean functions in B_0 can be computed using NAND gates.

Question 3: In our proof that there exist some functions on n bits that are not computable by circuits of size $(1-\varepsilon)\frac{2^n}{n}$, we considered gates with fan-in 2. What happens to the proof if we allow gates of fan-in 3?

Question 4: Prove the $\mathcal{RP} \subseteq \mathcal{BPP}$.

Question 5: Prove the $\mathcal{ZPP} = \mathcal{RP} \cap \mathsf{co}\mathcal{RP}$.