

## Homework 1

Students are welcome to work together, but *every student must write up their own solutions, independently!* I strongly encourage students to use LaTeX for writing up their solutions. Please see the course web-page for a template file.

Each question is worth 10 points.

**Question 1:**

1. Consider the following game. I roll a six-sided die and look at the outcome,  $v$ , but without revealing it to you. You make a guess  $g$ , after which I reveal some hint by choosing a random value  $h \neq g$  and revealing whether or not  $v = h$ . I then offer you a chance to change your guess. Clearly if I revealed that  $v = h$ , you would prefer to change your guess to  $h$ . What if  $v \neq h$ : is there any advantage in changing your guess? To answer the question, let  $V$  be a random variable denoting the value of the die, let  $G_1$  be a random variable denoting your original guess, let  $G_2$  be a random variable denoting your final guess, if you change it after receiving a hint, and let  $H$  be a random variable denoting the value of the hint. Evaluate the following probabilities:
  - (a)  $\Pr[H \neq V]$
  - (b)  $\Pr[G_1 = V \wedge H \neq V]$
  - (c)  $\Pr[G_1 = V \mid H \neq V]$
  - (d)  $\Pr[G_1 \neq V \mid H \neq V]$
  - (e)  $\Pr[G_2 = V \mid H \neq V]$
  - (f)  $\Pr[G_2 \neq V \mid H \neq V]$
2. Consider the following game. I roll a six-sided die and look at the outcome,  $v$ , but without revealing it to you. You make a guess,  $g$ , after which I reveal some hint,  $h$ , which is chosen randomly, subject to the constraint that  $h \neq g \wedge h \neq v$ . (Note, this is *not* the same method of choosing  $h$  as in the previous part!) I offer you a chance to change your guess. Is there any advantage to changing your guess? Justify your answer by calculating the following probabilities
  - (a)  $\Pr[H = h]$
  - (b)  $\Pr[G_1 = V \mid H = h]$  (hint: use the fact that this is equal to  $\frac{\Pr[G_1 = V \wedge H = h]}{\Pr[H = h]}$ )
  - (c)  $\Pr[G_2 = V \mid H = h]$  (hint: as above).

**Question 2:** Exercise 2.4 in the book

**Question 3:** Exercise 2.6 in the book.

**Question 4:** Exercise 2.9, parts (a) and (b), in the book. Hint for part (b): it may not be the most “natural” message space. But it has size greater than 26.