Introduction to Software Testing

Syntactic Logic Coverage (Ch. 8.2)

Software Testing & Maintenance

SWE 437/637

go.gmu.edu/SoftwareTestingFall24

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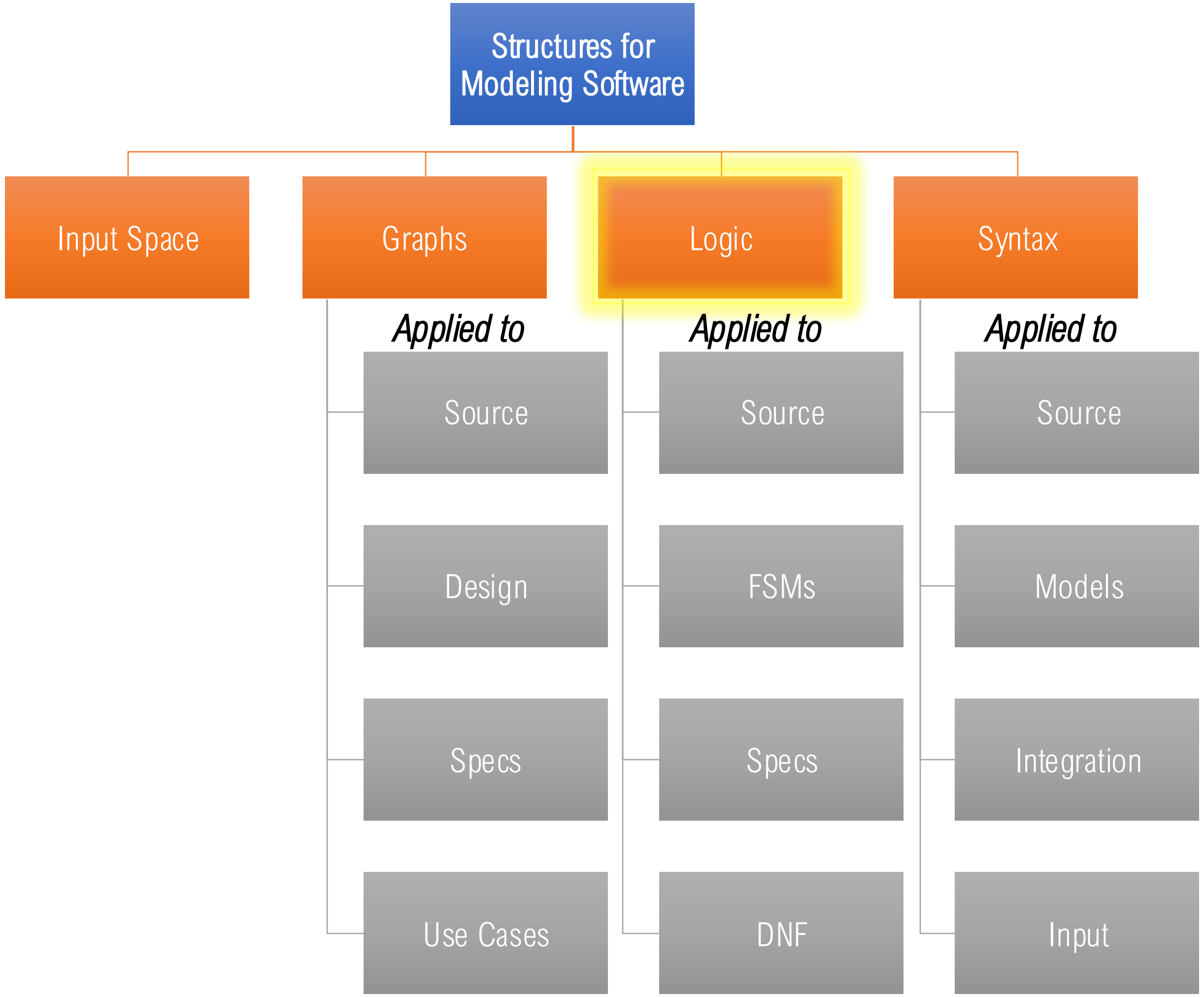
(Dr. B for short)

Logic Coverage



Disjunctive Normal Form (DNF)

Disjunctive Normal Form (DNF) is a common representation for Boolean functions

- Slightly different notation and terminology
- **Literal**: a clause or the negation of a clause (a, $\bar{a}$).
- **Term**: is a set of literals connected by logical operator *and* (∧), represented by adjacency.
  - $\boldsymbol{a} \wedge \boldsymbol{b}$ becomes $\boldsymbol{ab}$
  - $\neg\boldsymbol{a} \wedge \boldsymbol{b}$ becomes $\overline{\boldsymbol{a}}\boldsymbol{b}$
  - $\neg\boldsymbol{a} \wedge \neg\boldsymbol{b}$ becomes $\overline{\boldsymbol{ab}}$
- Terms are also called **implicants**, because if a single term is true, it implies that the entire predicate is true
- **Predicate**: a set of terms connected by or, which is represented

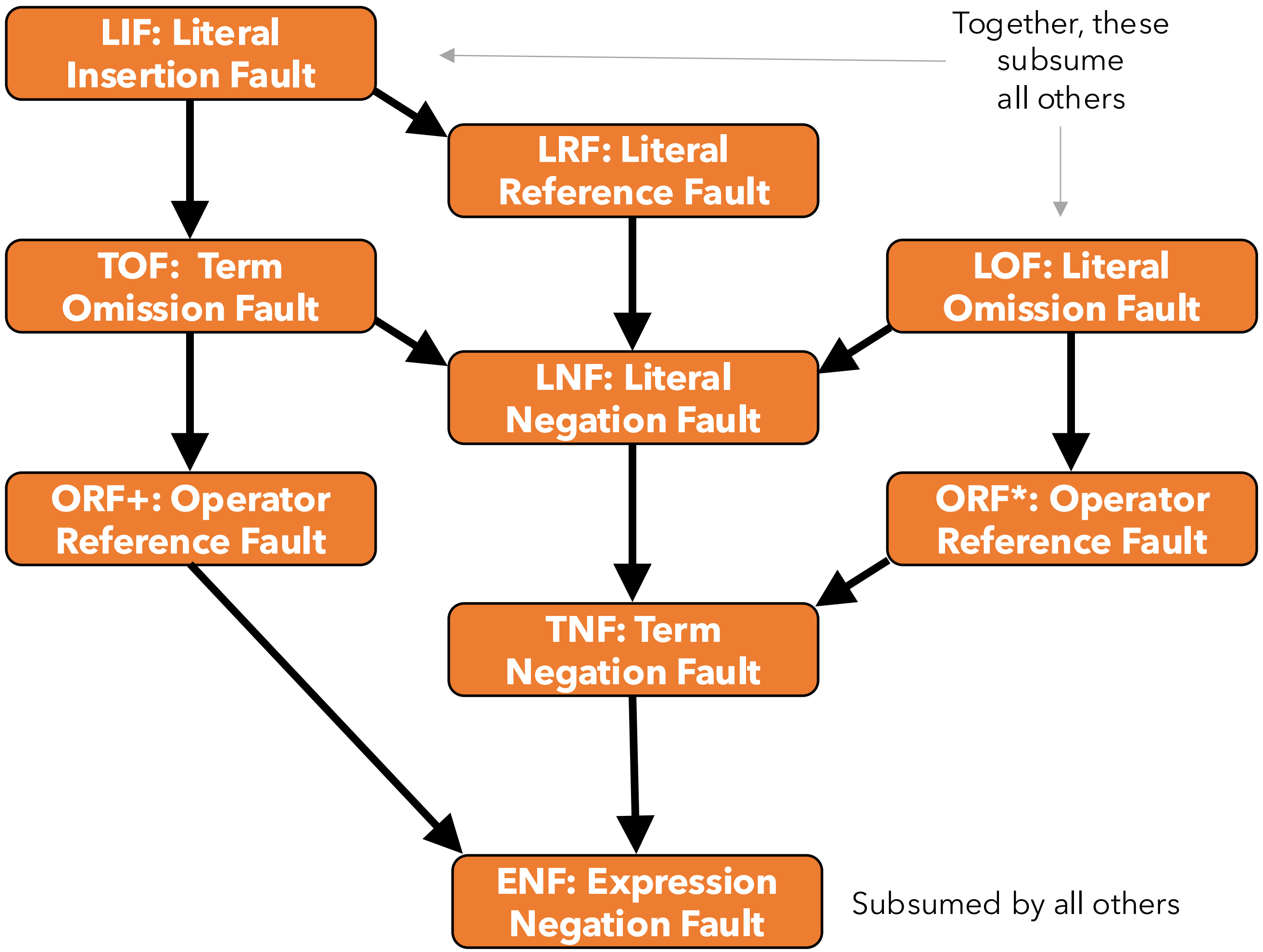
DNF Fault Classes

There are 9 types of syntactic faults on DNF predicates; we want

them.

Fault Class	Intended Expression	Faulty Expression
ENF: expression negation fault	$f = ab + c$	$f = \overline{ab + c}$
TNF: term negation fault	$f = ab + c$	$f = \overline{ab} + c$
TOF: term omission fault	$f = ab + c$	$f = ab$
LNF: literal negation fault	$f = ab + c$	$f = a\overline{b} + c$
LRF: literal reference fault	$f = ab + bcd$	$f = ad + bcd$
LOF: literal omission	$f = ab + c$	$f = a + c$

DNF Fault Class Subsumption


LIF: Literal Insertion Fault
LRF: Literal Reference Fault
TOF: Term Omission Fault
LOF: Literal Omission Fault
LNF: Literal Negation Fault
ORF+: Operator Reference Fault
ORF*: Operator Reference Fault
TNF: Term Negation Fault
ENF: Expression Negation Fault
Together, these subsume all others
Subsumed by all others

If we can find **LIF** and **LOF** faults, we will find *all* faults

Implicant Coverage

An obvious coverage thought is to make each implicant (term) evaluate to true

This only tes F negation of the entire

DEFINITION

Implicant Coverage (IC) – Given DNF representation of a predicate $\boldsymbol{f}$ and its negation $\bar{\boldsymbol{f}}$, for each implicant in $\boldsymbol{f}$ and $\bar{\boldsymbol{f}}$, ***TR*** contains the requirement that the implicant evaluate to true.

Examples: $f = ab + b\bar{c}, \bar{f} = \bar{b} + \bar{a}c$

Implicants: $\{ab,\ b\bar{c}, \bar{b}, \bar{a}c\}$

Possible test set: { TTF, FFT }

Improving on Implicant Coverage

Additional definitions:

Proper subterm: a term with one or more clauses removed

$\boldsymbol{abc}$ has proper subterms, $\boldsymbol{a},\ \boldsymbol{b},\ \boldsymbol{c}, \boldsymbol{ab}, \boldsymbol{ac}, \boldsymbol{bc}$

Prime implicant: an implicant such that no proper subterm is an implicant

Given $\boldsymbol{f} = \boldsymbol{ab} + \boldsymbol{a\bar{b}c}, \boldsymbol{ab}$ is a prime implicant, but $\boldsymbol{a\bar{b}c}$ is not, because proper subterm $\boldsymbol{ac}$ is an implicant (because the predicate can be simplified to $\boldsymbol{f} = \boldsymbol{ab} + \boldsymbol{ac}$, and we'll soon see how to determine that)

Redundant implicant: an implicant that can be removed without changing the value of the predicate

Given $\boldsymbol{f} = \boldsymbol{ab} + \boldsymbol{ac} + \boldsymbol{b\bar{c}}$, implicant $\boldsymbol{ab}$ is redundant because the

Simplifying Predicates

We can use Karnaugh maps (K-maps) to simplify DNF predicates

Given predicate $\boldsymbol{f = ab + ac + b\bar{c}}$

Group clauses into pairs (or one pair and one single clause) and populate the possible values of the clauses.

ab

c	00	01	11	10
0			t	
1			t	

Values use *Grey code* ordering (rather than binary counting) where only one truth value changes at a time across columns or down rows.

Populate the truth table where true values are listed as “t”; false values are (by convention) simply left blank.

Simplifying Predicates

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ab

		00	01	11	10
c	0			t	
	1			t	t

Values use Grey *code* ordering (rather than binary counting) where only one truth value changes at a time across columns or down rows

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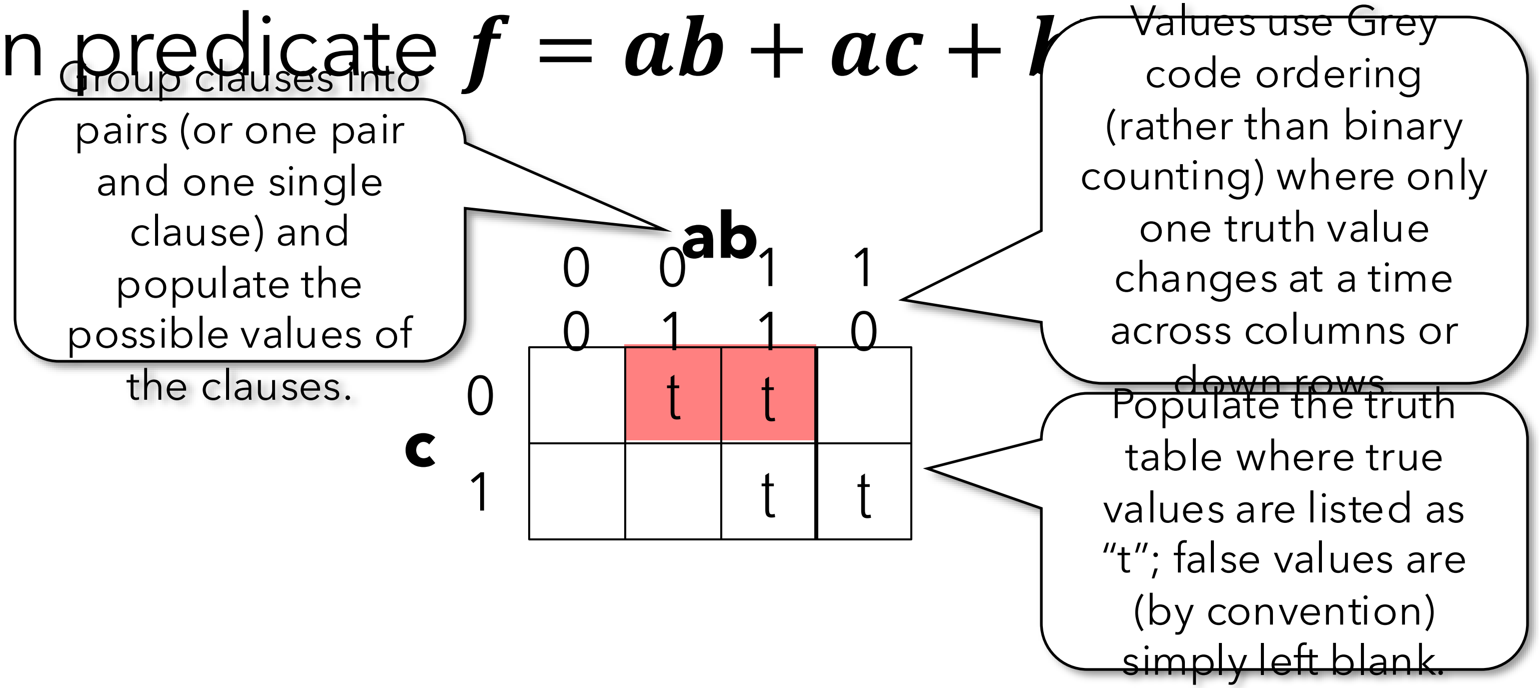

		ab			
		00	01	11	10
c	0		t	t	
	1			t	t

Group clauses into pairs (or one pair and one single clause) and populate the possible values of the clauses.

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Simplifying Predicates

We can use Karnaugh maps (K-maps) to simplify DNF predicates

Given predicate $\boldsymbol{f = ab + ac + b\bar{c}}$

Simplifies to $\boldsymbol{f = ac + b\bar{c}}$

ab

c	00	01	11	10
0		t	t	
1			t	t

Simplifying Predicates

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Given predicate $\boldsymbol{f = ab + ac + b\bar{c}}$

Simplifies to $\boldsymbol{f = ac + b\bar{c}}$

		ab			
		00	01	11	10
c	0		t	t	
	1			t	t

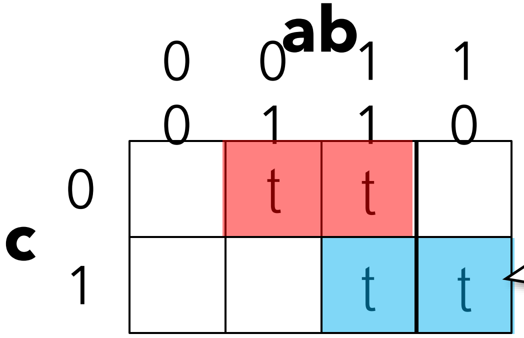
Select maximal rectangles in the table, sized 2^m by 2^n (1x1, 1x2, 2x2, 2x4, 4x4, 4x8, etc.); it's okay if they overlap

Simplifying Predicates

We can use Karnaugh maps (K-maps) to simplify DNF predicates

Given predicate $\boldsymbol{f} = \boldsymbol{ab} + \boldsymbol{ac} + \boldsymbol{b\bar{c}}$

Simplifies to $\boldsymbol{f} = \boldsymbol{ac} + \boldsymbol{b\bar{c}}$



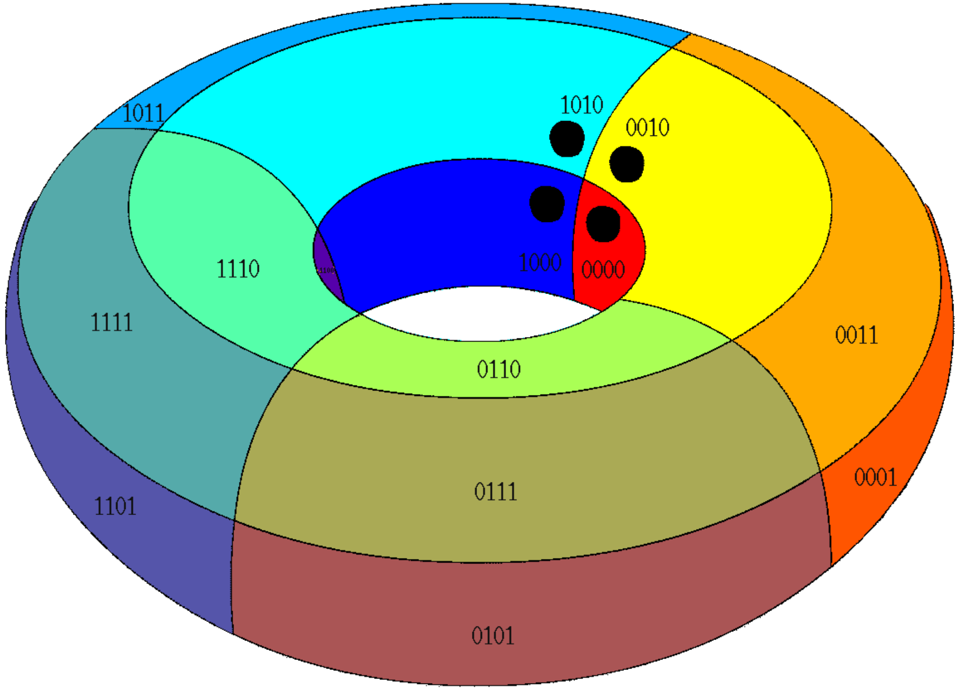
		ab			
		00	01	11	10
c	0		t	t	
	1			t	t

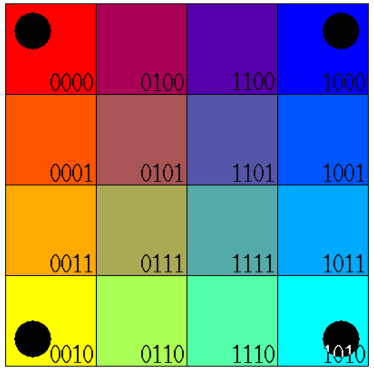
Select maximal rectangles in the table, sized 2^m by 2^n (1x1, 1x2, 2x2, 2x4, 4x4, 4x8, etc.); it's okay if they overlap

K-Maps are toroidal!

K-Maps are a torus, not a plane

- The bottom row wraps around to the top row
- The right column wraps around to the left column





● 0000	0100	1100	● 1000
0001	0101	1101	1001
0011	0111	1111	1011
● 0010	0110	1110	● 1010

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K-Maps are toroidal!

Given the predicate $\boldsymbol{f = \overline{bd}}$

Draw the K-map

ab

cd	00	01	11	10
00	t			t
01				
11				
10	t			t

K-Maps are toroidal!

Given the predicate $\boldsymbol{f = \overline{bd}}$

Draw the K-map

ab

cd	00	01	11	10
00	t			t
01				
11				
10	t			t

These 4 true values are a single 2x2 rectangle!

Prime Implicants

Given the predicate $f = abc + ab\bar{d} + \bar{a}bcd + a\bar{b}c\bar{d} + a\overline{cd}$

Draw the K-map

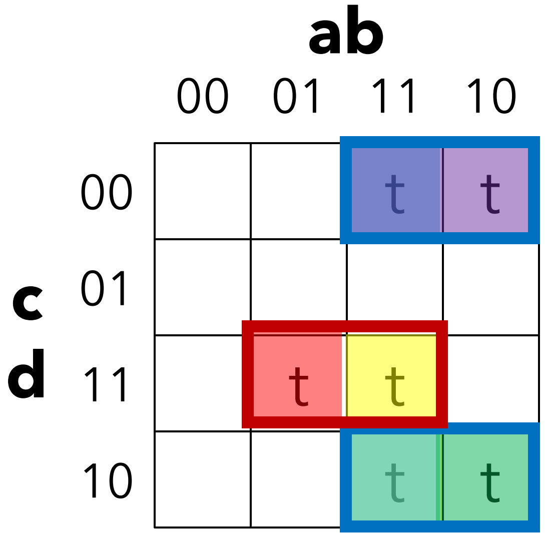
ab

		00	01	11	10
c d	00			t	t
	01				
	11		t	t	
	10			t	t

Prime Implicants

Given the predicate $f = abc + ab\bar{d} + \bar{a}bcd + a\bar{b}c\bar{d} + a\overline{cd}$

Draw the K-map



cd \ ab	00	01	11	10
00			t	t
01				
11		t	t	
10			t	t

Not prime implicants:

- $ab\bar{d}$ (part of $a\bar{d}$)
- $\bar{a}bcd$ (part of bcd)
- $a\bar{b}c\bar{d}$ (part of $a\bar{d}$)
- $a\overline{cd}$ (part of $a\bar{d}$)

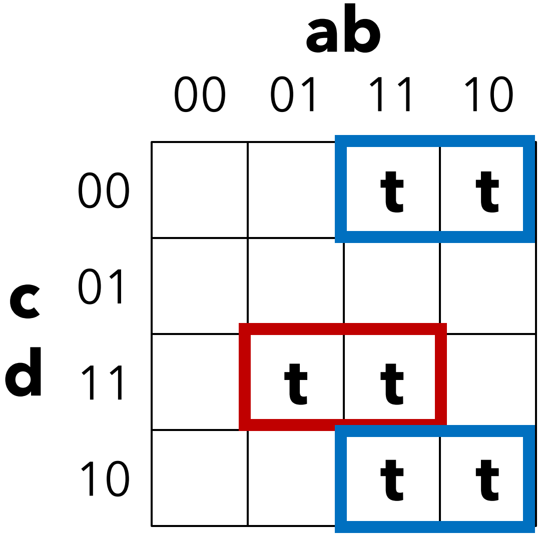
All these have proper subterms that are implicants

Minimal DNF representation: $f = a\bar{d} + bcd$

Minimal representation

A **minimal DNF representation** is one with only *prime*, *non-redundant* implicants

- Not minimal: $\boldsymbol{f = abc + ab\bar{d} + \bar{a}bcd + a\bar{b}c\bar{d} + a\overline{cd}}$
- Minimal (simplified) equivalent from previous slide: $\boldsymbol{f = a\bar{d} + bcd}$



	ab			
cd	00	01	11	10
00			**t**	**t**
01				
11		**t**	**t**	
10			**t**	**t**

Determination

Given predicate $f = b + \overline{ac} + ac$, suppose we want to identify when b determines f

Draw K-map

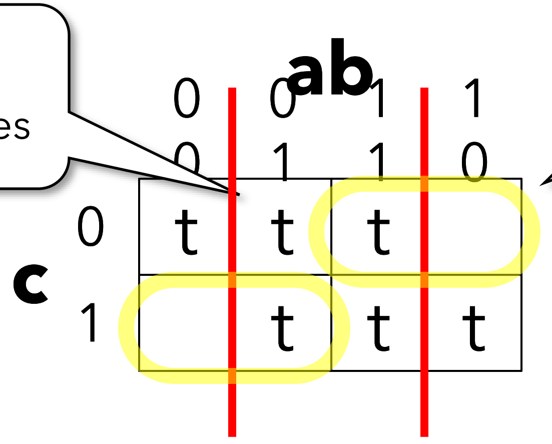
		ab			
		00	01	11	10
c	0	t	t	t	
	1		t	t	t

Determination

Given predicate $f = b + \overline{ac} + ac$, suppose we want to identify when b determines f

Draw K-map

Identify the boundaries where b changes value.



c \ ab	00	01	11	10
0	t	t	t	
1		t	t	t

If two cells adjacent to the boundary have different values for f, then b determines f for those two cells.

b determines f for $a\bar{c} + \bar{a}c$

Predicate Negation

Given predicate $f = ab + bc$, suppose we want to negate f

Draw the K-map for f.

ab

c	00	01	11	10
0			t	
1		t	t	

Negate all the cells in the K-map.

ab

c	00	01	11	10
0	t	t		t
1	t			t

Write down the result: $\overline{f} = \overline{b} + \overline{ac}$

True and False Points

Given $f = ab + cd$

cd \ ab	00	01	11	10
00			t	
01			t	
11	t	t	t	t
10			t	

True points □ are those cells in the K-map where the value of the predicate is true

□

False points are those where the value is false

Unique True Points (UTPs)

A **unique true point (UTP)** with respect to a given implicant is an assignment of truth values such that

- The given implicant is true
- All other implicants are false

Thus a unique true point test focuses on *only one* implicant

Unique True Points (UTPs)

Given $f = ab + cd$

ab

cd	00	01	11	10
00			t	
01			t	
11	t	t	t	t
10			t	

Unique true points for ab

TTFF, TTFT, TTTF

Unique true points for cd

FFTT, FTTT, TFTT

TTTT is a true point, but not a *unique* true point

Multiple UTP Coverage

A minimal representation guarantees the existence of at least one unique true point for each implicant.

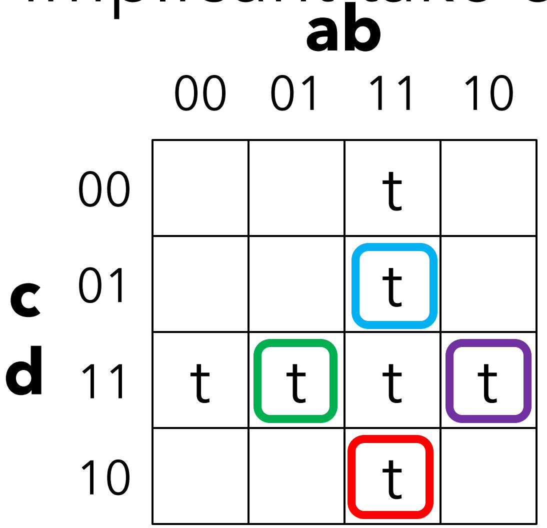
DEFINITION

Multiple Unique True Point Coverage (MUTP) – Given a minimal DNF representation of a predicate *f*, for each implicant *i*, choose unique true points (UTPs) such that clauses not in *i* are true and false.

Multiple UTP Coverage

Given $f = ab + cd$

Choose unique true points for each implicant such that literals not in the implicant take on values true and false



cd \ **ab**	00	01	11	10
00			t	
01			t	
11	t	t	t	t
10			t	

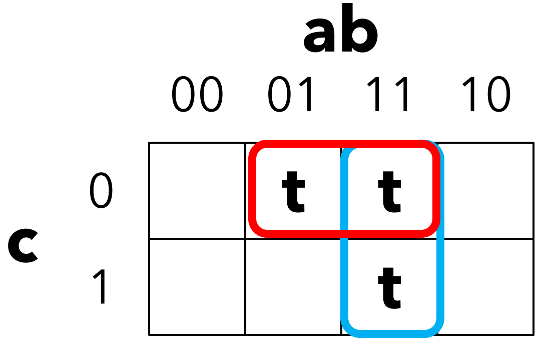
For implicant ab, choose **TTFT** and **TTTF**

For implicant cd, choose **FTTT** and **TFTT**

MUTP Infeasibility

Given the predicate $\boldsymbol{f} = \boldsymbol{ab} + \boldsymbol{b\bar{c}}$

- Implicants are { $\boldsymbol{ab}, \boldsymbol{b\bar{c}}$ }
- Both implicants are prime
- Neither implicant is redundant

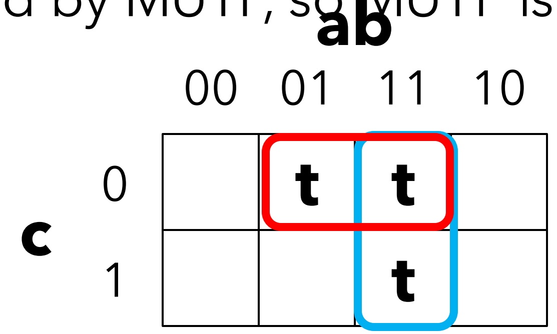


c \ ab	00	01	11	10
0		t	t	
1			t	

MUTP Infeasibility

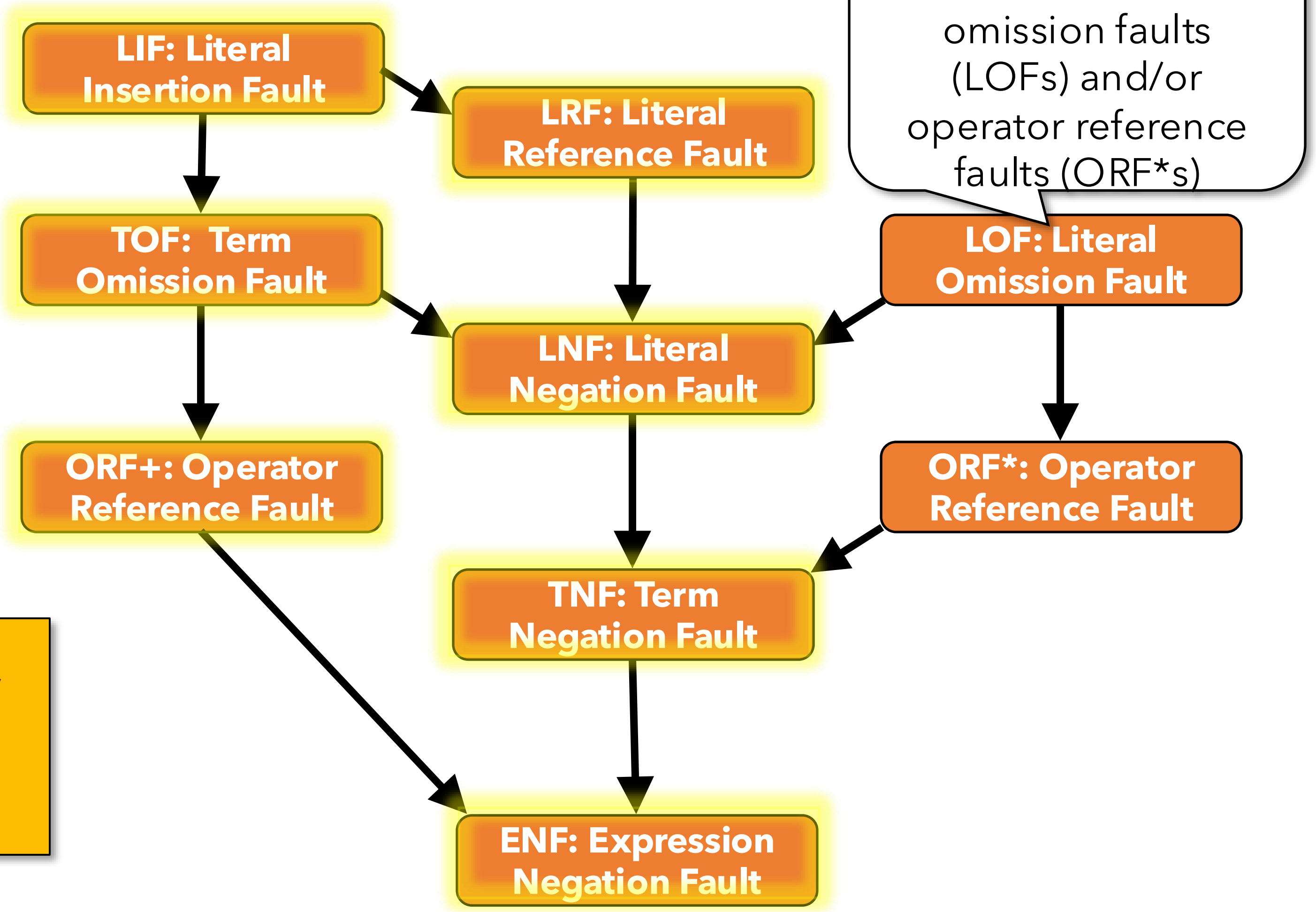
Unique true points required by MUTP

- $\boldsymbol{ab}$: {TTT} causes $\boldsymbol{ab}$ to be true and $\boldsymbol{b\bar{c}}$ to be false
  - But there's no way to also make clause c both true and false while keeping the implicants true and false as required by MUTP, so MUTP is infeasible
- $\boldsymbol{b\bar{c}}$: {FTF} causes $\boldsymbol{ab}$ to be false and $\boldsymbol{b\bar{c}}$ to be true
  - But there's no way to also make clause a both true and false while keeping the implicants true and false as required by MUTP, so MUTP is infeasible



	00	01	11	10
0		t	t	
1			t	

MUTP Fault Detection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TNF: Term Negation Fault
ENF: Expression Negation Fault

Now we need a way to find all literal omission faults (LOFs) and/or operator reference faults (ORF*s)

When feasible, MUTP finds all *literal insertion faults* (LIFs)